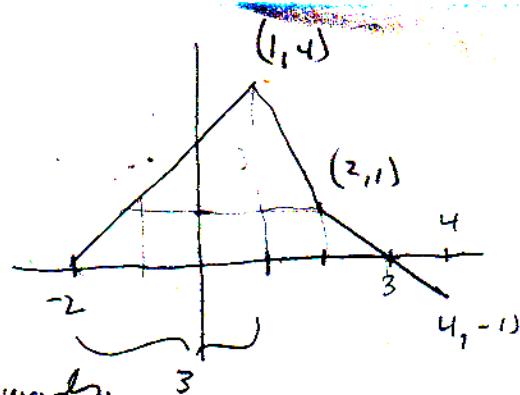


I

a) $g(4) = \int_1^4 f(t) dt$

$\frac{1}{2} \cdot 3 \cdot 1 + 1 + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{2}$

$g(-2) = -\frac{1}{2} \cdot 3 \cdot 4 = -6$ ^{2pts} *why because $\int_{-2}^1 f(t) dt$ backwards.*



b) $g'(1) \Rightarrow f(1) = 4$ from graph.
because $g'(x) = f(x)$ by ^{not} FTC.

c) g' $\begin{array}{c} + \quad 0 \quad - \\ | \quad | \quad | \\ -2 \quad 3 \quad 4 \end{array}$ $(2, -6)$ is a min in g' ,
 $(3, 3)$ is a max g' test,

$g(3) = \int_1^3 f(t) dt = \frac{1}{2} \cdot 3 \cdot 1 + 1 + \frac{1}{2} = 3$

$(4, \frac{5}{2})$ is a min dec to it.

d) $g'' = f'$ $\begin{array}{c} + \quad 0 \quad - \\ | \quad | \quad | \\ -2 \quad 1 \quad 2 \end{array}$ The 2nd der dNE when the slope of f DNE and this happens when slope doesn't become smooth change it "sharp corners".

e) $x=1$ is a pI because

g changes from concave up to down.

$x=2$ is not a pI because g is concave down on both sides.

Neither 2 or 6

II

- a) $\begin{cases} 0 \\ 3 \leftarrow \\ 6 \leftarrow \\ 9 \leftarrow \\ 12 \leftarrow \\ 15 \leftarrow \\ 18 \leftarrow \\ 21 \leftarrow \\ 24 \end{cases}$

width = 6

$6(10.4 + 11.2 + 11.3 + 10.2)$

$6(43.1)$

258.6

In 24 hours approximately 258.6 gallons of water flowed through the pipe.

II

- b) $R(t)$ is continuous and differentiable for $[0, 24]$
 $R(0) = R(24) = 9.6$ $\therefore$ by Rolle's Thm there exists
 ξ such that $R(\xi) = 0$, so yes that point exists.
 Explained but no mention of
 Rolle's Thm = -1
 Example = -5

c) $Q(t) = \frac{1}{79} (768 + 23t - t^2)$

$$\frac{1}{24} \int_0^{24} \frac{1}{79} (768 + 23t - t^2) dt = 10.785$$

water flowed at 10.785 gal per hour.

- III a) ξ_i is sample point in the section $[x_{i-1}, x_i]$. ξ_i is usually
 picked the Left, Right or mid pt. at sections
 b) $x_i - x_{i-1}$ is width of i th section. The larger of these is called
 the mesh. as the mesh $\rightarrow 0$ the error $\rightarrow 0$
 c) $f(\xi_i)$ is the height of the i th sample rectangle. when
 multiplied by the width of the section we get the area
 of the rectangle.
 d) $f(\xi_i)(x_i - x_{i-1})$ area of i th rectangle. These are summed to
 approximate the area under the curve
 e) mesh is the largest of the sections. see details under "b".

Extra.

Let N = number of people in audience

- $.20N$ = heard 60 minutes
 $.10N$ = heard 20 minutes
 $.135N$ = heard 20 min
 $.135N$ = heard 40 min

total N people heard
 $60(.12N) + 0(.10N) + 20(.35N)$
 $+ 40(.35N)$

$$12N + 0 + 7N + 14N$$

$33N$ minutes of
 the speech

so the average minutes heard
 was $\frac{33N}{N} = 33$ minutes.

2/16/00

$$\begin{array}{r} 100 \quad 0 \\ \hline 90 \quad 7 \\ \hline 80 \quad 5 \\ \hline 70 \quad 5 \\ \hline 60 \quad 3 \\ \hline 59 \quad 1 \\ \hline \end{array}$$

99

55

1686



21

80.3

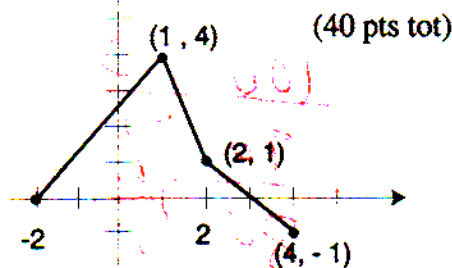
1. What is the purpose of the study?
 2. What are the research questions?
 3. What is the significance of the study?
 4. What are the limitations of the study?
 5. What are the conclusions of the study?

Chapters 7 & 8 Exam

I The graph of the function f , consisting of three line segments, is given:

$$\text{Let } g(x) = \int_1^x f(t) dt.$$

- a) Compute $g(4)$, $g(-2)$. Be very clear.
 b) Find $g'(1)$. *Explain.*
 c) Find the coordinates of and identify all extreme values of g on the closed interval $[-2, 4]$. Justify your answer.
 d) Explain how you know the second derivative of g is not defined at $x=1$ and $x=2$.
 e) Which (if any) of the values $x=1$ and $x=2$ the x -coordinates of points of inflection of the graph of g ? Justify your answer.



II The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table shows the rate as measured every 3 hours for a 24-hour period. (40 pts tot)

- a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t) dt$.
 b) *Explain the meaning of the statement $R'(t) = 0$ in terms of water flow.* Using correct units, explain the meaning of your answer in terms of water flow.

- c) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

- d) The rate of water flow $R(t)$ can be approximated by

$$Q(t) = \frac{1}{79}(768 + 23t - t^2). \text{ Use } Q(t) \text{ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.}$$

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

III The definite integral of a function f over an interval $[a, b]$ is denoted by $\int_a^b f(x) dx$ and defined as follows: $\int_a^b f(x) dx = \lim_{\text{mesh} \rightarrow 0} f(X_i)(x_i - x_{i-1})$.

Describe each of the following and its purpose:

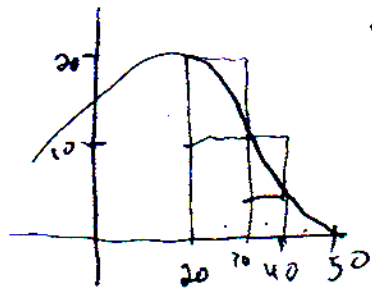
(20 pts tot)

- a) X_i b) $x_i - x_{i-1}$ c) $f(X_i)$ d) $f(X_i)(x_i - x_{i-1})$ e) mesh

Extra Credit ----- 5 pts -----

A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk, and ten percent slept through the entire talk. Half the remainder heard one-third of the talk and the other half heard two-thirds of the talk. What was the average number of minutes of the talk heard by members of the audience?

I



$$a) 10 \cdot 20 + 10 \cdot 20 + 10 \cdot 10$$

$$200 + 200 + 100$$

$$500$$

$$b) 10 \cdot 15 + 10 \cdot 15 + 10 \cdot 0$$

$$150 + 150 + 0$$

$$300$$

$$c) A = \frac{400 + 200}{2} = 300$$

d) $f(x)$ has max when $x=50$ because $f'(50)=0$ is critical value. $f' > 0$ for $x < 50$ and $f' < 0$ for $x > 50$ $\therefore$ max; f' test.

E)

$$\int_a^b f(x) dx = F(b) - F(a)$$

we want $F(50)$

$$\int_{20}^{50} f(x) dx = F(50) - F(20)$$

$$F(50) = \int_{20}^{50} f(x) dx + F(20)$$

$$300 + 150 = 450$$

From Part C Given

II

$$f(x) = \frac{1}{x+1} \quad x_i = \frac{i}{N} \quad x_i = \frac{i-1}{N} \text{ and } i=1, 2, \dots, N.$$

$$\sum_{i=1}^N f(x_i) (x_i - x_{i-1})$$

$$f(x_i) = \frac{1}{\frac{i-1}{N} + 1}$$

$$= \frac{1}{\frac{i-1+N}{N}} = \frac{N}{i-1+N}$$

$$\sum_{i=1}^N \frac{N}{i-1+N} \cdot \frac{1}{N} =$$

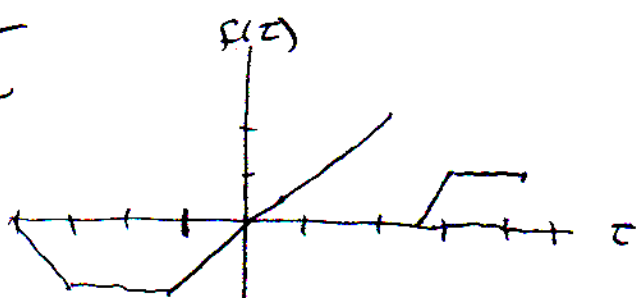
$$\sum_{i=1}^N \frac{1}{i-1+N}$$

$$x_i - x_{i-1}$$

$$\frac{i}{N} - \frac{i-1}{N}$$

$$\frac{i-i+1}{N} = \frac{1}{N}$$

III



$$g(x) = \int_{-1}^{2x} f(t) dt$$

$$a) -4 \leq t \leq 2 \quad \text{or} \quad \frac{5}{2} \leq t \leq \frac{9}{2}$$

$$-4 \leq 2x \leq 2 \quad \frac{5}{2} \leq 2x \leq \frac{9}{2}$$

$$-2 \leq x \leq 1 \quad \frac{5}{4} \leq x \leq \frac{9}{4}$$

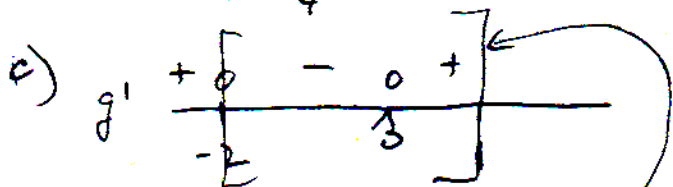
b) $g'(x) = 1$

$$g'(x) = f(2x) \cdot 2 \stackrel{\text{Set}}{=} 1$$

$$f(2x) = \frac{1}{2}$$

$2x \stackrel{\text{Set}}{=} \frac{1}{2}$ from y value, this is a
 $\uparrow$ this is an x value

$$x = \frac{1}{4}$$



$$g'(1) = 2f(1) = 2 \cdot 1 = 2 > 0$$

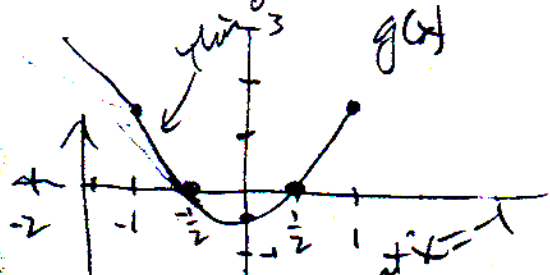
$$\int_{-1}^{2x} f(t) dt \stackrel{\text{Set}}{=} 0 \text{ for } x \text{ interval}$$

this = zero when $\int_{-1}^1$ from graph.

$$2x = 1 \quad x \text{ intercept } (\frac{1}{2}, 0)$$

$$x = \frac{1}{2}$$

also $\frac{1}{2} \leq 2x \leq 1$ so $2x = \frac{1}{2}$
 $x = \frac{1}{4}$



this is straight line
 that der does exist at $x=1$

must use $\pi - x$.
 Since $\pi - x$ is
 within the range of
 $\sin^{-1}$ fct and x is not,

$g(x)$ is the area trapped by $f(t)$ from
 $t = -1$ to where $2x$ is but t only
 goes to 2 when there is a break,
 since the 2nd interval does not include
 $t = -1$; it is not part of the domain of
 g because FTC requires fct name to be
 continuous.

$$g'(x) = 2f(2x) \stackrel{\text{Set}}{=} 0$$

$$f(2x) = 0 \quad 2x = -4 \quad x = -2$$

$$g(-2) = \int_{-1}^{-4} f(t) dt = - \int_{-4}^{-1} f(t) dt = - (f(2, t)) = \frac{5}{2}$$

$$g(0) = \int_{-1}^0 f(t) dt = -\frac{1}{2}$$

max $(-\frac{1}{2}, \frac{5}{2})$ end pt
 dec from it.

$$g(1) = \int_{-1}^2 f(t) dt = -\frac{1}{2} + 2 = \frac{3}{2}$$

max $(1, \frac{3}{2})$ end pt
 inc to it.

$$x \text{ intercept } (-\frac{1}{2}, 0)$$

extra, $f(x) = \int_0^{\sin x} \sin^{-1} t dt$

$$f'(x) = \sin^{-1}(\sin x) \cos x$$

since $x \in 2^{\text{nd}}$ quad;
 $\sin x = \sin(\pi - x)$ so

$$\sin^{-1}(\sin(\pi - x)) \cos x$$

$$(\pi - x) \cos x$$

2001

100	1
90	5
80	7
70	5
60	7
50	2

2082

27

77.1

100

↓

45

Good exptm.

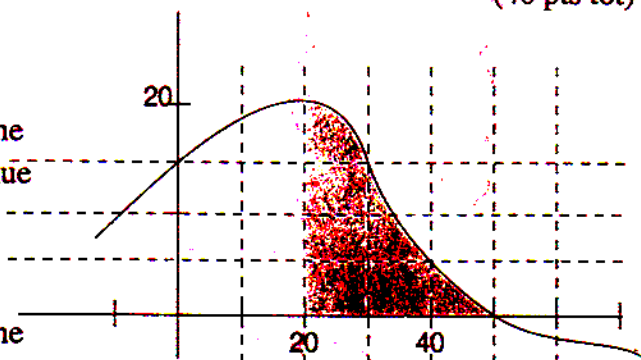
Hard to grade because
usually either they
saw it and got it or they
didn't.

Chapters 7 & 8 Exam

I

The graph on the right is the derivative of $F(x)$ (i.e. This is the graph of $F'(x)$.) $F(x)$ is some function which is continuous and differentiable and $F(20) = 150$. (40 pts tot)

- a) Partition the interval $[20, 50]$ into 3 sections of equal length. Choose the left edge of the section for the sample point and write the approximating sums for the shaded area under the curve $F'(x)$. Compute the value of the sum.
- b) Partition the interval $[20, 50]$ into 3 sections of equal length. Choose the right edge of the section for the sample point and write the approximating sums for the shaded area under the curve $F'(x)$. Compute the value of the sum.
- c) Let the approximation of the shaded area be the average of your answers to parts a and b. Identify this average by calling it "A".
- d) For what x value does $F(x)$ have a maximum? Explain!
- e) Using the First FTC, determine an approximation for that maximum value of $F(x)$. (hint: Write the first FTC down, re-read this problem, look at your work, then work out the answer to this problem.)



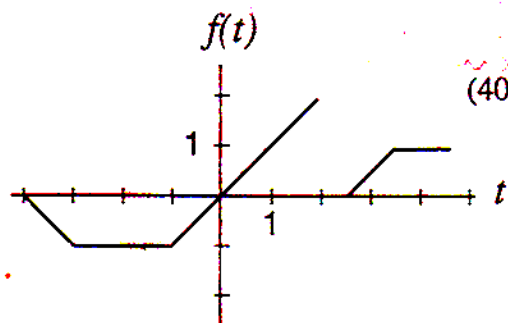
II

Let $f(x) = \frac{1}{x+1}$, $x_i = \frac{i}{n}$, $X_i = \frac{i-1}{n}$, and $i = 1, 2, \dots, n$. (20 pts)

Let the points form a partition of the closed interval $[0, 1]$. Use summation notation to write the corresponding approximating sum. While keeping the sigma notation, simplify completely while removing the letters f , X , and x .

III

Let $g(x) = \int_{-1}^{2x} f(t) dt$ when given the graph of $y = f(t)$: (40 pts tot)



- a) Explain clearly why the domain of $g(x)$ is: $D_g: -2 \leq x \leq 1$. Include reasoning how you know there is not a second interval.
- b) For what values of x does $g'(x) = 1$?
- c) Determine the coordinates of all extrema and all axis intercepts for the function $g(x)$.
- d) Draw $g(x)$.

Extra Credit ----- 5 pts -----

Given: $\frac{\pi}{2} < x < \pi$. Find $f'(x)$ if $f(x) = \int_0^{\sin x} \sin^{-1} t \, dt$.

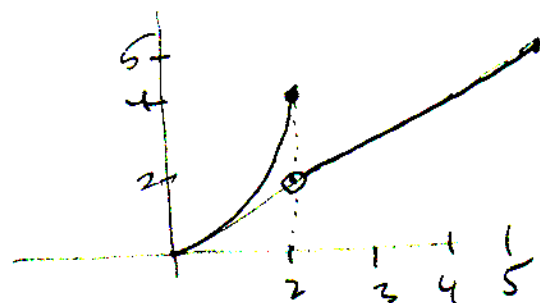
- I a) ξ_i Sample point in section $[x_{i-1}, x_i]$, ξ_i is usually picked to be left, right or center of section.
- b) $x_i - x_{i-1}$ is the width of a section. The largest is called the mesh. As the mesh $\rightarrow 0$, the section decreases in width to yield exact area.
- c) $f(\xi_i)$ height of current sample rectangle. When multiplied by the width of the section, we get the area of the rectangle.
- d) $f(\xi_i)(x_i - x_{i-1})$ area of the i th rectangle. These are summed to approximate the area under the curve.
- e) mesh The largest of all sections in the interval, when the mesh $\rightarrow 0$ the approximation becomes the exact area.

						Left midrule	Right rule
II	Seconds	1	2	3	4	5	
20pts	mph	16	40	62	82	100	
						$0 + 16 + 40 + 62 + 82$	$16 + 40 + 62 + 82 + 100$
						200	300
	$\frac{\text{miles}}{\text{hour}}$					$\frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}}$	$100 \left(\frac{5280}{3600} \right) \approx 147 \text{ feet}$
							The estimates differ by 147 feet.

- III
- a) $\int_0^1 \frac{1}{x} dx$ not defined in the interval
- b) $\int_0^{\infty} x dx$ FTC requires numbers a and b ∞ is not a number
- c) $\int_0^{\pi} \tan x dx$ $\tan$ not defined at $\frac{\pi}{2}$ not continuous.
- d) $\int_{-1}^2 5x dx$ f is not defined for $x < 0$

IV

15pts $f(x) = \begin{cases} x^2 & 0 \leq x < 2 \\ x & 2 \leq x \leq 5 \end{cases}$



b) $\int_0^2 x^2 dx + \int_2^5 x dx$

$\frac{x^3}{3} \Big|_0^2 + \frac{x^2}{2} \Big|_2^5$

$\frac{8}{3} + \frac{25}{2} - 2$

$\frac{2}{3} + \frac{25}{2}$

$\frac{4 + 75}{6} = \frac{79}{6} \approx 13.167$

Q
15 pts $\int_0^x f(t) dt = \frac{2x}{4+x^2}$ a must take der to get $f(x)$
10 pts

$$f(x) = \frac{(4+x^2)2 - 2x(2x)}{(4+x^2)^2}$$

$$= \frac{8+2x^2-4x^2}{(4+x^2)^2}$$

$$f(x) = \frac{8-2x^2}{(4+x^2)^2} \geq 0$$

$$x \geq \pm 2$$

$$= \frac{8-2x^2}{(4+x^2)^2}$$

Zeros of f
occur when
 $x \in \{-2, 2\}$

$$f(0) = \frac{8-0}{4^2} = \frac{1}{2}$$

Q
15 pts f is der of $\int_0^x f(t) dt$ and f' is obviously the der of f .

curve b could be $-\sin 2x = \int_0^x f(t) dt$

Notice period
does not
change,

curve b could be $-2\cos 2x = f(x)$

curve a could be $48\sin 2x = f'(x)$

also notice Amplitude
Does change.

when $b=0$ a and c have extreme
Values. when $c < 0$ a is concave

EXTRA

$$\lim_{x \rightarrow 3} \left(\frac{1}{x-3} \int_3^x \frac{\sin t}{t} dt \right)$$

denom so
2nd derivatives $\left(\int_3^x \frac{\sin t}{t} dt \right)' =$
 $a'' = c$ or $c'' = a$

$$\lim_{x \rightarrow 3} \frac{\int_3^x \frac{\sin t}{t} dt}{x-3}$$

b increases
zero when $a=0$
so $b' = a$

$$\left. \frac{\sin t}{t} \right|_{t=3} =$$

$$\lim_{x \rightarrow 3} \frac{\int_3^x \frac{\sin t}{t} dt - \int_3^3 \frac{\sin t}{t} dt}{x-3}$$

$$\frac{1}{3} \sin 3.$$

$$\approx .047$$

1st Def of Der of $\int_3^x \frac{\sin t}{t} dt$
evaluated at
 $x=3$

100	0
90	14
80	4
70	3
60	2
59	1

99
↓
52

$$\frac{2055}{24} = 85.6$$

The needs to be replaced with
one of the "weapons" type problems.

Section I was poor. The problem here was I wanted $g'(4) = \frac{2x}{4+x^2}$
but I couldn't get it
the way I did it.

Section II could use some work too.

This should be replaced with one of the AP Problems
of similar nature.

Section III was a good idea but with only 10 PTS.

100 100

100	1
90	6
80	6
70	5
60	
50	1

157

$$\frac{1586}{19} = 83.5$$

Greater than...

Copy the original problem.

Convince me that **you** understand the concept.

Per

Date

2002

(Remember: 3 decimal places!!)

CHAPTER #7 & 8

I The definite integral of a function f over an interval $[a, b]$ is denoted by $\int f(x) dx$ and defined

$$\text{as follows: } \int f(x) dx = \lim_{\text{mesh} \rightarrow 0} f(X_i)(x_i - x_{i-1}).$$

Describe each of the following **and** its purpose:

(15 pts tot)

- a) X_i b) $x_i - x_{i-1}$ c) $f(X_i)$ d) $f(X_i)(x_i - x_{i-1})$ e) mesh

II A racing car achieves the following speeds (in miles per hour)

during the first five seconds of a race. Use appropriate left and right sums to approximate the distance the driver travels during the first five seconds. By how many feet do they differ?

(20 points)

seconds	1	2	3	4	5
mph	16	40	62	82	100

III For each of the following, explain why we cannot evaluate the definite integral.

(5 pts ea)

- a) $\int_0^1 \frac{1}{x} dx$ b) $\int_0^{\infty} x dx$ c) $\int_0^{\pi} \tan x dx$ d) $\int_{-1}^2 \sqrt{x} dx$

IV A function f is **piecewise continuous** on $[a, b]$ if it is continuous except, possibly, for a finite set of points at which it has a discontinuity. It can be shown that if f is **piecewise continuous**on $[a, b]$, then $\int_a^b f(x) dx$ exists. Let $f(x) = \begin{cases} x^2 & ; 0 \leq x \leq 2 \\ x & ; 2 < x \leq 5 \end{cases}$ (15 pts tot)

- a) Sketch the graph of f .
 b) Write the definite integrals needed to describe the area of the region bounded by the graph of f and the x -axis between $x = 0$ and $x = 5$.
 c) Find the area described.

$$g(x) = \int_0^x f(t) dt \quad \text{and}$$

V Assume f is a continuous function and that $\int_0^x f(t) dt = \frac{2x}{4+x^2} = f(x)$ (15 pts tot)

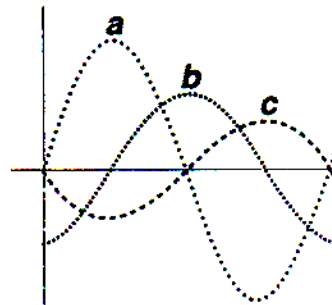
Poor question

- a) Determine $f(0)$ b) Find the zeros of f' , if any.

VI The figure shows the graphs of f , f' , and $\int_0^x f(t) dt$.

Identify each graph and explain your choices.

(15 pts)



EXTRA CREDIT

5 pts

$$\text{Evaluate: } \lim_{x \rightarrow 3} \left(\frac{1}{x-3} \int_3^x \frac{\sin t}{t} dt \right)$$

$$I \quad f(x) = \int_2^x \sqrt{t^3 + 1} \, dt$$

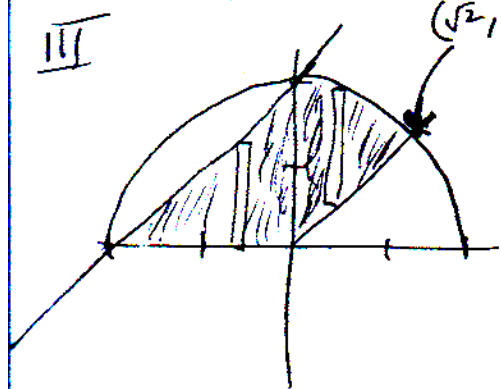
$$f(2) = 0 \quad f'(x) = \sqrt{x^3 + 1} \Big|_{x=2} = \sqrt{8+1} = 3$$

$$m = 3 \quad (2, 0)$$

$$y - 0 = 3(x - 2)$$

$$y = 3x - 6$$

III



$$x^2 + y^2 = 4 \quad y > x \quad y = x + 2$$

$$x^2 + x^2 = 4$$

$$2x^2 = 4$$

$$x = \pm \sqrt{2}$$

$$\int_{-2}^0 x+2 \, dx + \int_0^{\sqrt{2}} \sqrt{4-x^2} - x \, dx$$

$$II \quad \int_2^5 2f(x) + 3 \, dx = 17$$

$$2 \int_2^5 f(x) \, dx + \int_2^5 3 \, dx = 17$$

$$2 \int_2^5 f(x) \, dx + 3 \times \left|_2^5 \right| = 17$$

$$+ 15 = 6 =$$

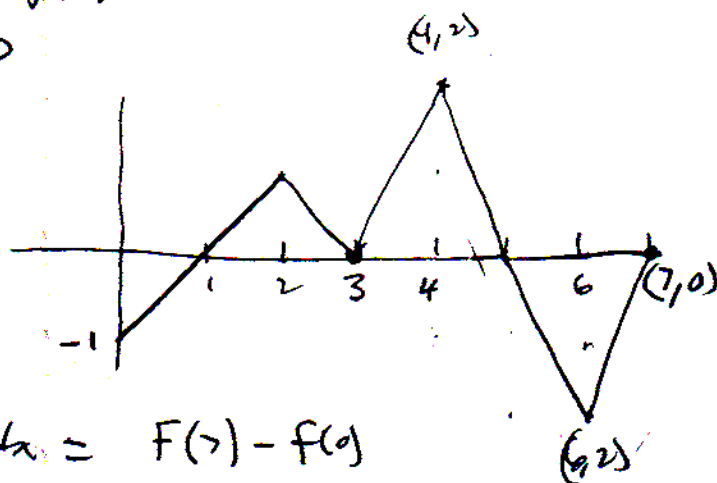
$$2 \int_2^5 f(x) \, dx = 8$$

$$\int_2^5 f(x) \, dx = 4$$

IV

$$f'(x) = \begin{cases} 1 & 0 < x < 2 \\ -1 & 2 < x < 3 \\ 2 & 3 < x < 4 \\ -2 & 4 < x < 6 \\ 1 & 6 < x < 7 \end{cases}$$

$$f(3) = 0$$



$$b) \quad f(0) = -1$$

$$f(7) = 0$$

$$c) \quad \int_0^7 f'(x) \, dx = F(7) - f(0)$$

$$0 - (-1)$$

using area rule
Rectangles

$$2 - 1 + 2 - 4 + 2 = 1$$

V

$$\int_0^{50} v(t) \, dt$$

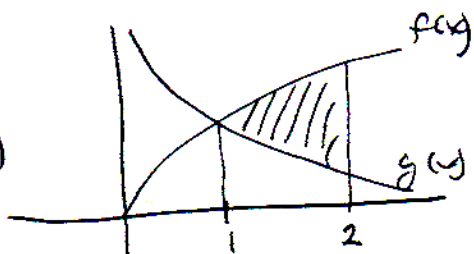
$$10(12) + 10(30) + 10(70) + 10(81) + 10(60)$$

$$\begin{array}{r} 120 \\ 300 \\ 700 \\ 810 \\ 600 \\ \hline 2530 \end{array}$$

This integral is the total distance
traveled in feet during the
time from zero to 50 seconds.

VI

a)



$$\int_0^2 \sqrt{x} dx - \int_0^2 \frac{1}{x} dx$$

$$\frac{2x^{3/2}}{3/2} - \ln x \Big|_0^2$$

Extra $4\sqrt{\frac{2}{3}} - \frac{2}{3} - \ln 2$

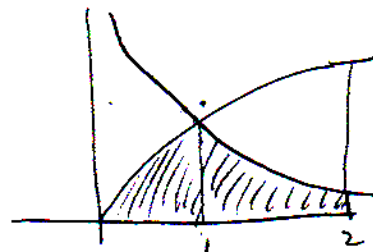
$$f(x) = \int_0^{\sin x} \sin^{-1}(t) dt$$

$$f'(x) = \sin^{-1}(\sin x) \cos x$$

$$\sin^{-1}(\sin(\pi - x)) \cos x$$

$$(\pi - x) \cos x$$

b)



$$\int_0^1 \sqrt{x} dx + \int_1^2 \frac{1}{x} dx$$

$$\frac{2x^{3/2}}{3/2} \Big|_0^1 + \ln x \Big|_1^2$$

$$\frac{2}{3} + \ln 2$$

$\frac{\pi}{2} < x < \pi$ is 2nd Quad.

since $x \in 2^{nd}$ Quad.

$$\sin(\pi - x) = \sin x$$

$$\sin \pi \cos x - \sin x \sin \pi$$

$$0 - (-\sin x)$$

$$\sin x$$

	4	5	T
100	0	1	1
90	3	3	6
80	4	5	9
70	3	3	6
60	0	0	0
59	5	6	11

100



24

$$\begin{array}{r} 1069 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 1323 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 2392 \\ \hline 33 \end{array}$$

71.3

73.5

72.5

Copy original problem.

Convince me that you understand the concept!

Per

Date

2003

No Calculators, of course.

Chapters 7 & 8 Exam

I Given: $f(x) = \int_2^x \sqrt{t^3 + 1} dt$. Find the equation of the tangent line at $x = 2$. (10 pts)

II Given: $\int_2^5 (2f(x) + 3) dx = 17$. Find $\int_2^5 f(x) dx$. (10 pts)

III Given the three equations: $x^2 + y^2 = 4$, $y = x + 2$, and $y = x$. (20 pts total)

A) Sketch the three equations on the same axis. Shade the region above the x -axis bounded by these curves.

B) Set up, but **Do Not Evaluate** the definite integral(s) which describe the shaded region in part A. Give a very explicit explanation of your methods.

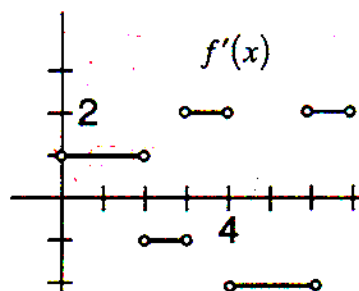
IV Assume $f'(x)$ is given by the graph on the right. (20 pts)

Suppose f is continuous and that $f(3) = 0$

a) Sketch a graph of f .

b) Find $f(0)$ and $f(7)$.

c) Find $\int_0^7 f'(x) dx$ in two different ways.



V The velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown in a table of values for $v(t)$, at 5 second intervals of time.

Approximate the definite integral, $\int_0^{50} v(t) dt$ with a Riemann sum,

using the mid-points of five subintervals of equal length.

Using correct units, explain the meaning of this integral. (20 pts)

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

VI Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$. Draw the graph showing the area computed and compute the exact area described. (tot 20)

a) $\int_1^2 f(x) dx - \int_1^2 g(x) dx$

b) $\int_0^1 f(x) dx + \int_1^2 g(x) dx$

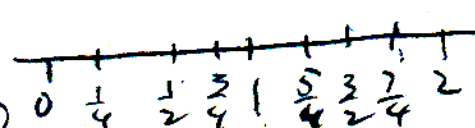
Extra Credit 5 pts

Given: $\frac{\pi}{2} < x < \pi$. Find $f'(x)$ if $f(x) = \int_0^{\sin x} \sin^{-1} t dt$.

# 1 R/C Section 7.3 # 2 Do pg 265 1 - 8 ALSO write a complete explanation of telescoping sums. Use pblm 8 in your demo. answers to pblm 8 are: $2^{100} - 1, \frac{-99}{100}, \frac{-100}{101}, \frac{101}{102}$	# 3 Pg 265 9, 11 - 16 hint on pblm 9: $x_n = 4 \text{ and } x_0 = 0$ # 4 R/C Pg 250 - 253 then R/C pg 255 - 260 1/28	# 5 Pg 253 - 254 1, 2, 6, 7 ----- # 6 Pg 276 - 277 1 - 14 Pblm 14 is important. Do a complete job on it! 1/29	# 7 Rewrite "Summary" IN YOUR OWN WORDS. Pg 278 - 279 ----- # 8 GQ Pg 279-280 1, 2, 3 RE Pg 280-281 1 - 6 Go thru #6 pblm by pblm in class to force student focus on being prepared. 1/30	RE Pg 281 7 - 15 # 9 ----- ALSO do "matching" on the side chalk board # 10
# 11 Pg 287 - 288 1 - 19 for pblm 14: Make Reimann Sums using 3 sections. Show intermediate values. Use FnInt(to compare area values. For pblms 15, 16, 17: <i>also</i> evaluate using FnInt(	# 12 Pg 291 - 292 10 - 18 2/4	R/C Pg 292-297 # 13 ----- # 14 Pg 297 1 - 12, 14 - 17 Important Lecture 2/5	R/C Pg 298 - 300 # 15 ----- Pg 300 - 301 1 - 30 , 32 # 16 Remember to include the "+ c " indefinite integral 2/6	# 17 , 18, 19 "Honor Quiz" 2/7
Holiday # 20 "Calculator 101" 2/10	MAA test # 21 GQ C8 Pg 302 1, 3, 4, 6, 7, 9 GQ Ch 1-8 Pg 302-303 1, 2, 3 2/11	# 22 "Semi Circle the Wagons" 2/12	# 23 RE Ch 1-8 Pg 304-306 1 - 3, 5 - 7, 9, 12 - 14, # 24 Pg 304-306 32-36, 38-40, 43, 50, 52 - 56 Upper limit on # 40 should be $\sqrt{\pi}$. 2/13	# 25, #26 "Translating ..." # 27 $Ff'f'f'(un)ctions$ 2/14
Holiday 2/17	Put last year's exam on the boards. 2/18	Exam # 1 Quarter 3 2003 2/19	Advanced Placement Calculus AB Chapters 7 & 8 First Exam Third Quarter 2/20 2/21	

- I
- Δx_i sample point in section $[x_{i-1}, x_i]$. Depending on the $f(x)$, this changes the approx from over to under or vice versa
 - $x_i - x_{i-1}$ width of a section. The largest section is called the mesh. As the mesh $\rightarrow 0$, this section decreases in width to yield Δx_i
 - $f(x_i)$ height of current rectangle, area is height times width at the particular section.
 - $f(x_i)(x_i - x_{i-1})$ area of the i^{th} rectangle. These are summed to approx the area under the curve
 - mesh The largest of all sections in the interval. as the mesh shrinks to zero all sections also shrink to zero ending up by summing line lengths to get area.

II

a) 

① $6 \left[\frac{1}{2} \left(\frac{1}{4} \right)^2 + \frac{1}{2} \left(\frac{3}{4} \right)^2 + \frac{1}{2} \left(\frac{5}{4} \right)^2 + \frac{1}{2} \left(\frac{7}{4} \right)^2 \right]$
 $3 \left[\left(\frac{1}{4} \right)^2 + \dots \right]$
 $\frac{3}{16} [1 + 3^2 + 5^2 + 7^2]$

② $f(x_i) = \left(\frac{2x_i - 1}{4} \right)^2$
 $x_i - x_{i-1} = \frac{1}{2}$

③ $\frac{3}{16} \sum_{i=1}^4 (2i-1)^2$

b) $\int_0^2 6t^2 dt = \frac{2(2)^3 - 2(0)^3}{2 \cdot 3} = \frac{16}{3}$
 16 feet

III

$$g(x) = x f(x) - \int_0^x f(t) dt$$

$$g'(x) = f(x) - x f'(x) - f(x)$$

$$= -x f'(x) \stackrel{?}{=} 0$$

$x \neq 0 \therefore f'(x) \stackrel{?}{=} 0$

$$\begin{array}{c} + \quad - \\ - \quad 0 \quad + \\ \hline 0 \quad \quad x \end{array}$$

If $x > 0$, max, if $x < 0$, min
 Thus the extremum occurs when $f'(x) = 0$

100	2
90	9
80	7
70	0
60	2
50	1

$\frac{1855}{21} = 88.3$
 without the 50
 90.3

102
 50

This was 1 question
 Short of what it
 Should have been
 make part I worth 10pts
 and ~~other~~ ^{new section worth} 15pts can be
 a graph and shows
 cross sections type.

10pts
 15pts

Copy the original problem.

Convince me that you understand the concept.

Per

Date

CHAPTER #7 & 8

I

The definite integral of a function f over an interval $[a, b]$ is denoted by $\int_a^b f(x) dx$ and definedas follows: $\int_a^b f(x) dx = \lim_{\text{mesh} \rightarrow 0} \sum_{i=1}^n f(X_i)(x_i - x_{i-1})$. Describe each of the following and its purpose? (25 pts tot)

- a) X_i b) $x_i - x_{i-1}$ c) $f(X_i)$ d) $f(X_i)(x_i - x_{i-1})$ e) mesh

II

A rocket travels at a speed of $6t^2$ feet per second t seconds after its projection.

- a) Obtain an approximating sum for the distance traveled by this rocket in the first two seconds, using the partition of the interval $[a, b]$ into four equal sections and the midpoint of each section as its sampling point.
- Express the result in expanded form of the sum. (5 pts)
 - Specifically state the value of $f(X_i)$. Specifically state the value of $(x_i - x_{i-1})$. (10 pts)
 - Express the result in a simplified summation notation. (5 pts)
- b) Using the first Fundamental Theorem of Calculus, compute the exact distance traveled by this rocket in the first two seconds. (10 pts)

III

Suppose f is a differentiable function. Show that if $x \neq 0$, that any maximum or minimum of $g(x) = xf(x) - \int_0^x f(t) dt$ occurs when $f'(x) = 0$. Explain. (20 pts)

IV

Let f be a function that is even and continuous over $[-3, 3]$. The function and its derivatives have the properties indicated in the table:

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

- a) Find the x -coordinate of any max or min. Identify. Justify of course.
- b) Find x -coordinate of any P.I.

c) Sketch.

(25 pts total)

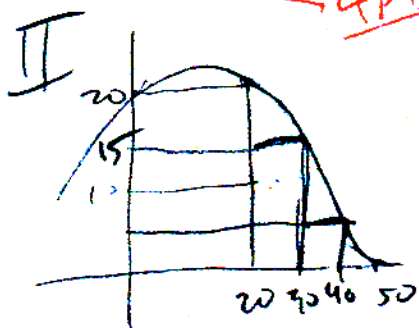
EXTRA CREDIT

5 pts

V

Find $f'(x)$ if $f(x) = \int_0^{\sin x} \sin^{-1} t dt$ $\frac{\pi}{4} < x < \pi$.

I Oil flows at $v = f(t)$ gal/min. If it is short length of time then $f(t) dt$ is amount of oil during that time. Sum up all these quantities to get total oil. Start at $t=60$ and go to $t=120$ to get the second hour.



a) $10(20) + 10(15) + 10(5)$
 $200 + 150 + 50 = 400$

c) $A = \frac{400 + 200}{2}$

$A = 300$

b) $10(15) + 10(5) + 10(0)$
 $150 + 50 = 200$

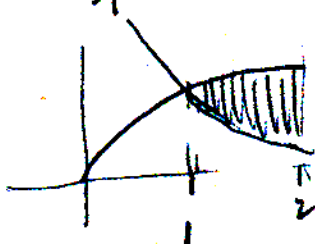
d) f' has a max at $x=50$ because $f' \geq 0$ at $x=50$, f is increasing as $x \rightarrow 50^-$ and f decreases ($f'(x)$ is negative) for $x > 50$. f has max at $x=50$ f' test.

e) $\int_{20}^{50} f(x) dx = F'(50) - F'(20)$

$F'(50) = F'(20) + \int_{20}^{50} f(x) dx$
 $= 150 + 300 = 450.$

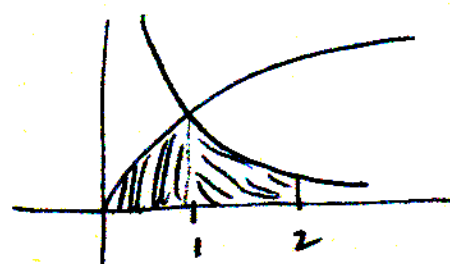
III

a) $\int_1^2 \sqrt{x} dx - \int_1^2 \frac{1}{x} dx$



$\frac{2}{3} x^{3/2} \Big|_1^2 - \ln x \Big|_1^2$
 $\frac{2\sqrt{2}}{3} - \frac{2}{3} - \ln 2 + \ln 1$
 $\frac{4\sqrt{2}}{3} - \frac{2}{3} - \ln 2$
 $\frac{4\sqrt{2}-2}{3} - \ln 2 = .526$

b) $\int_0^1 \sqrt{x} dx + \int_1^2 \frac{1}{x} dx$



$\frac{2}{3} x^{3/2} \Big|_0^1 + \ln x \Big|_1^2$
 $\frac{2}{3} + \ln 2 = 1.36$

IV

a) $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$
 $-\frac{1}{2} \cos 2x \Big|_0^{\frac{\pi}{2}}$
 $-\frac{1}{2} (\cos \pi - \cos 0)$
 $-\frac{1}{2} (-1 - 1)$
 1

b) $\int_0^{\frac{\pi}{2}} x \sin x^2 \, dx$
 $-\frac{1}{2} \cos x^2 \Big|_0^{\frac{\pi}{2}}$ *Calc must be in Radians!*
 $-\frac{1}{2} \left[\cos \frac{\pi^2}{4} - 1 \right]$
 $-\frac{1}{2} (\cos 2467 - 1)$
 $-\frac{1}{2} (-.7812 - 1) = .891$

c) $\int_0^1 e^{\sqrt{x}} \, dx$
 $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} \Big|_0^1$
 $2e - 2e - (0 - 2)$
 2

d) $\int_0^1 (2x+1)^2 \, dx$
 $\int_0^1 4x^2 + 4x + 1 \, dx$
 $\frac{4}{3}x^3 + 2x^2 + x \Big|_0^1$
 $\frac{4}{3} + 2 + 1 = \frac{13}{3}$

e) $\int_1^4 \frac{1+x}{x} \, dx$
 $\int_1^4 \frac{1}{x} + 1 \, dx$
 $\ln x + x \Big|_1^4$
 $\ln 4 + 4 - \ln 1 - 1$
 $2 \ln 2 + 3$
 4.368

f) $\int_1^e \ln x \, dx$
 $x \ln x - x$
 $e - e - (1 \cdot 0 - 1)$
 1

Extra.

To generate the Volume of a sphere, set up an integral of a cross sections which would be a circle. limits of the integral could be from $-r$ to r . the radius of the "typical cross section" would be $\sqrt{r^2 - x^2}$.

The area of the cross section is πr^2 which becomes $\pi(r^2 - x^2)$.

$$\pi \int_{-r}^r (r^2 - x^2) \, dx = \pi \left(rx - \frac{x^3}{3} \right) \Big|_{-r}^r$$

$$\pi \left(r^2 - \frac{r^3}{3} - \left(-r^2 + \frac{r^3}{3} \right) \right) = \frac{4\pi r^3}{3}$$

Excellent room.

Excellnt RGM.
a wording change as indicated
would be an improvement

I invested on starting sacrifically that

$f(t)$ it was (rate)(short time) = small volume
and that all the "small volumes"
added would be the integral.

most tried to say "The der of an
integrated is the rate" and That's
what we have". I say That's wrong
what you are trying to justify

100	0
90	2
80	7
70	6
60	3
50	2

98

56

1557 77.85

20

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Name

CALC 78

Per

Date

94

Chapters 7 & 8 Exam

I

Oil is leaking out of a ruptured tanker at rate of $v = f(t)$ gallons per minute. Supply the reasoning that the following definite integral expresses the total quantity of oil which leaked out of the tanker during the second hour after the rupture. Be very clear but do not use specific examples.

$$\begin{array}{l} \text{volume} \\ \text{of} \\ \text{oil} \end{array} = \int_{60}^{120} f(t) dt$$

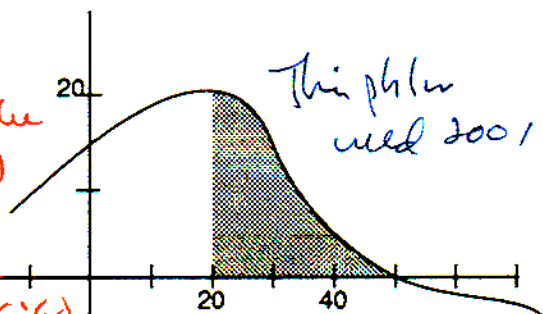
(20 pts)

II

The graph on the right is the derivative of $F(x)$ (i.e. This is the graph of $F'(x)$.) $F(x)$ is some function which is continuous and differentiable and $F(20) = 150$.

(30 pts tot)

- a) Partition the interval $[20, 50]$ into 3 sections of equal length. Choose the left edge of the section for the sample point and write the approximating sums. Compute the value of the sum. *for the shaded area under the curve $f'(x)$*
- b) Partition the interval $[20, 50]$ into 3 sections of equal length. Choose the right edge of the section for the sample point and write the approximating sums. Compute the value of the sum. *for the shaded area under the curve $f'(x)$*
- c) Let the approximation of the shaded area be the average of your answers to parts a and b. Identify this average by calling it "A".
- d) For what x value does $F(x)$ have a maximum? Explain!
- e) Using the First FTC, determine an approximation for the maximum value of $F(x)$. (hint: Write the first FTC down, re-read this problem, look at your work, then work out the answer to this problem.)



III

Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$. Draw the graph showing the area computed and compute the exact area described.

(tot 20)

a) $\int_1^2 f(x) dx - \int_1^2 g(x) dx$

b) $\int_0^1 f(x) dx + \int_1^2 g(x) dx$

IV

Compute the value of the definite integrals. A calculator may be of use.

(5 pts ea)

a) $\int_0^{\pi/2} \sin 2x dx$

b) $\int_0^{\pi/2} x \sin x^2 dx$

c) $\int_0^1 e^{\sqrt{x}} dx$

d) $\int_0^1 (2x+1)^2 dx$

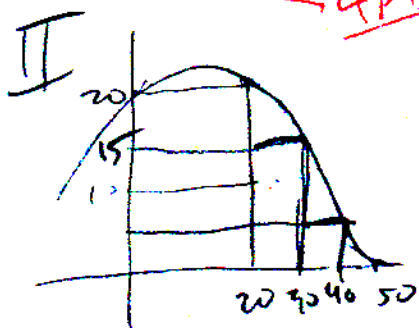
e) $\int_1^4 \frac{1+x}{x} dx$

f) $\int_1^e \ln x dx$

Extra Credit ----- 5 pts -----

Explain how you could use a definite integral to generate the formula for the volume of a sphere which had radius r .

I Oil flows at $v = f(t)$ gal/min. If it is short length of time then $f(t) dt$ is amount of oil during that time. Sum up all these quantities to get total oil. Start at $t=60$ and go to $t=120$ to get the second hour.



a) $10(20) + 10(15) + 10(5)$
 $200 + 150 + 50 = 400$

c) $A = \frac{400 + 200}{2}$

$A = 300$

b) $10(15) + 10(5) + 10(0)$
 $150 + 50 = 200$

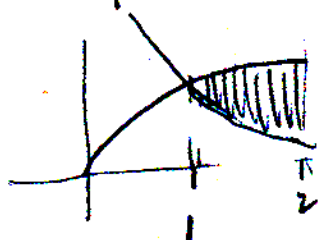
d) f' has a max at $x=50$ because $f' \geq 0$ at $x=50$, f is increasing as $x \rightarrow 50^-$ and f decreases ($f'(x)$ is negative) for $x > 50$. f has max at $x=50$ f' test.

e) $\int_{20}^{50} f(x) dx = F'(50) - F'(20)$

$F'(50) = F'(20) + \int_{20}^{50} f(x) dx$
 $= 150 + 300 = 450.$

III

a) $\int_1^2 \sqrt{x} dx - \int_1^2 \frac{1}{x} dx$



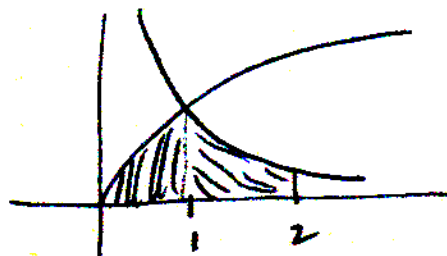
$\frac{2}{3} x^{3/2} \Big|_1^2 - \ln x \Big|_1^2$

$\frac{2\sqrt{2}}{3} - \frac{2}{3} - \ln 2 + \ln 1$

$\frac{4\sqrt{2}}{3} - \frac{2}{3} - \ln 2$

$\frac{4\sqrt{2}-2}{3} - \ln 2 = .526$

b) $\int_0^1 \sqrt{x} dx + \int_1^2 \frac{1}{x} dx$



$\frac{2}{3} x^{3/2} \Big|_0^1 + \ln x \Big|_1^2$

$\frac{2}{3} + \ln 2 = 1.36$

IV

$$a) \int_0^{\frac{\pi}{2}} \sin 2x \, dx$$

$$-\frac{1}{2} \cos 2x \Big|_0^{\frac{\pi}{2}}$$

$$-\frac{1}{2} (\cos \pi - \cos 0)$$

$$-\frac{1}{2} (-1 - 1)$$

$$1$$

$$b) \int_0^1 (2x+1)^2 \, dx$$

$$\int_0^1 4x^2 + 4x + 1 \, dx$$

$$\frac{4}{3} x^3 + 2x^2 + x \Big|_0^1$$

$$\frac{4}{3} + 2 + 1 = \frac{13}{3}$$

$$c) \int_0^{\frac{\pi}{2}} x \sin x^2 \, dx$$

$$-\frac{1}{2} \cos x^2 \Big|_0^{\frac{\pi}{2}}$$

Calc must be in Radians!

$$-\frac{1}{2} \left[\cos \frac{\pi^2}{4} - 1 \right]$$

$$-\frac{1}{2} (\cos 2467 - 1)$$

$$-\frac{1}{2} (-.7812 - 1) = .891$$

$$d) \int_1^4 \frac{1+x}{x} \, dx$$

$$\int_1^4 \frac{1}{x} + 1 \, dx$$

$$\ln x + x \Big|_1^4$$

$$\ln 4 + 4 - \ln 1 - 1$$

$$2 \ln 2 + 3$$

$$4.368$$

$$2e - 2e - (0 - 2)$$

$$2$$

$$e) \int_1^e \ln x \, dx$$

$$x \ln x - x$$

$$e - e - (1 \cdot 0 - 1)$$

$$1$$

Extra.

To generate the Volume of a sphere, set up an integral of a cross sections which would be a circle. limits of the integral could be from $-r$ to r . the radius of the "typical cross section" would be $\sqrt{r^2 - x^2}$.

The area of the cross section is πr^2 which becomes $\pi(r^2 - x^2)$.

$$\pi \int_{-r}^r (r^2 - x^2) \, dx = \pi \left(rx - \frac{x^3}{3} \right) \Big|_{-r}^r$$

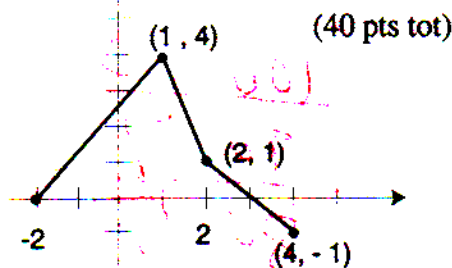
$$\pi \left(r^2 - \frac{r^3}{3} - \left(-r^2 + \frac{r^3}{3} \right) \right) = \frac{4\pi r^3}{3}$$

Chapters 7 & 8 Exam

I The graph of the function f , consisting of three line segments, is given:

$$\text{Let } g(x) = \int_1^x f(t) dt.$$

- a) Compute $g(4)$, $g(-2)$. Be very clear.
 b) Find $g'(1)$. *Explain.*
 c) Find the coordinates of and identify all extreme values of g on the closed interval $[-2, 4]$. Justify your answer.
 d) Explain how you know the second derivative of g is not defined at $x=1$ and $x=2$.
 e) Which (if any) of the values $x=1$ and $x=2$ the x -coordinates of points of inflection of the graph of g ? Justify your answer.



II The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table shows the rate as measured every 3 hours for a 24-hour period. (40 pts tot)

- a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t) dt$.

Using correct units, explain the meaning of your answer in terms of water flow.

- b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

- c) The rate of water flow $R(t)$ can be approximated by

$Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

III The definite integral of a function f over an interval $[a, b]$ is denoted by $\int_a^b f(x) dx$ and defined

$$\text{as follows: } \int_a^b f(x) dx = \lim_{\text{mesh} \rightarrow 0} f(X_i)(x_i - x_{i-1}).$$

Describe each of the following and its purpose:

(20 pts tot)

- a) X_i b) $x_i - x_{i-1}$ c) $f(X_i)$ d) $f(X_i)(x_i - x_{i-1})$ e) mesh

Extra Credit ----- 5 pts -----

A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk, and ten percent slept through the entire talk. Half the remainder heard one-third of the talk and the other half heard two-thirds of the talk. What was the average number of minutes of the talk heard by members of the audience?

2/16/00

$$\begin{array}{r} 100 \quad 0 \\ \hline 90 \quad 7 \\ \hline 80 \quad 5 \\ \hline 70 \quad 5 \\ \hline 60 \quad 3 \\ \hline 59 \quad 1 \end{array}$$

99

55

1686



21

80.3

1. What is the purpose of the study?
 2. What are the research questions?
 3. What is the study design?
 4. What is the sample size?
 5. What are the inclusion and exclusion criteria?
 6. What are the variables?
 7. What are the data collection methods?
 8. What are the data analysis methods?
 9. What are the results?
 10. What are the conclusions?

- I
- ξ_i sample point in section $[x_i - x_{i-1}]$ depending on the function, this changes approx from over to under or vice versa
 - $(x_i - x_{i-1})$ section, The largest section is called the mesh, we take the limit as the mesh $\rightarrow 0$ to get exact area
 - $f(\xi_i)$ height of current rectangle, rectangle area is found by multiplying this height by the width of the sections.
 - telescoping sum, when the summation is expanded the section values add and subtract $x_1 - x_0 + x_2 - x_1 + \dots + x_n - x_{n-1}$ which adds to $x_n - x_0$

II

$$f(x) = \frac{1}{x+1} \quad x_i = \frac{i}{N} \quad \xi_i = \frac{i-1}{N} \quad i = 1, 2, \dots, N$$

$$\sum_{i=1}^N f(\xi_i) (x_i - x_{i-1})$$

$$\sum_{i=1}^N \frac{1}{1 + \frac{i-1}{N}} \cdot \frac{1}{N}$$

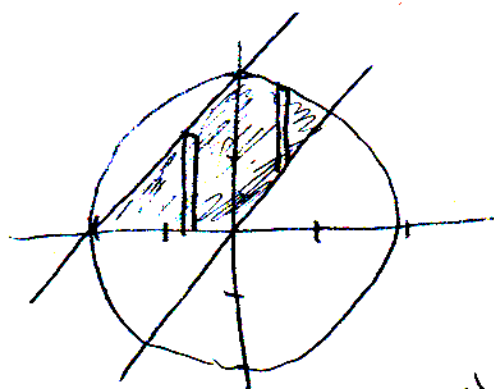
$$\sum_{i=1}^N \frac{1}{i-1+N}$$

$$f(\xi_i) = \frac{1}{\frac{i-1}{N} + 1} = \frac{1}{\frac{i-1+N}{N}} = \frac{N}{i-1+N}$$

$$x_{i-1} = \frac{i-1}{N}$$

$$x_i - x_{i-1} = \frac{i}{N} - \frac{i-1}{N} = \frac{1}{N}$$

III



$$\int_{-2}^0 x+2 \, dx + \int_0^{\sqrt{2}} \sqrt{4-x^2} - x \, dx$$

$$\begin{aligned} x^2 + y^2 &= 4 \\ x^2 + x^2 &= 4 \\ 2x^2 &= 4 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

IV

$$\begin{aligned} G(x) &= f'(x) \\ H(x) &= G'(x) \end{aligned}$$

$$f(0) = 0$$

$$f(1) = 1$$

$$G(0) = 0$$

$$G(1) = 0$$

$$H(0) = 2$$

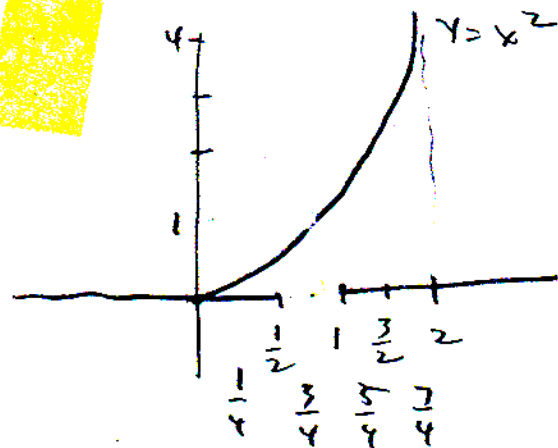
$$H(1) = 1$$

$$a) \int_0^1 G(x) \, dx = F(1) - f(0) = 1 - 0 = 1$$

"the der of what is G"

$$b) \int_0^1 H(x) \, dx = G(1) - G(0) = 0 - 0 = 0$$

V



partitions at $0, \frac{1}{2}, 1, \frac{3}{2}, 2$

mesh = $\frac{1}{2}$

sample pts $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$

$$\left(\frac{1}{4}\right)^2 \cdot \frac{1}{2} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{2} + \left(\frac{5}{4}\right)^2 \cdot \frac{1}{2} + \left(\frac{7}{4}\right)^2 \cdot \frac{1}{2}$$

$$\frac{1}{2} \sum_{i=1}^4 \left(\frac{2i-1}{4}\right)^2 = \frac{1}{32} \sum_{i=1}^4 (2i-1)^2$$

Extra $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{\frac{3}{8}} \Big|_{-1}^2 = -\frac{3}{8}$

Since $f(x) = \frac{1}{x^4}$ is not continuous in the interval $[-1, 2]$ the $-\frac{3}{8}$ has no meaning. Certainly the area is positive - it is infinite!

85 NS 5000

5000

100

97-2

95

74

12-2

89

84-3

80

75

65

57.

$$\frac{1201}{15} = 80.06$$

Good exam, most did
a good job. I'm
satisfied.

Copy original problem.

Convince me that you understand the concept!

Chapters 7 & 8 Exam

I

Briefly explain the meaning and use of each of the following terms:

(30 pts total)

- a) X_i b) $(x_i - x_{i-1})$ c) $F(X_i)$ d) telescoping sum

II

Let $F(x) = \frac{1}{x+1}$, $x_i = \frac{i}{n}$, $X_i = \frac{i-1}{n}$ and $i = 1, 2, \dots, n$.The points form a partition of the closed interval $[0, 1]$.

Use summation notation to write the corresponding approximating sum.

While keeping the sigma notation, simplify to remove the letters F , X , and x .

(15 pts)

III

Given the three equations: $x^2 + y^2 = 4$, $y = x + 2$, and $y = x$.

(15 pts total)

- A) Sketch the three equations on the same axis. Shade the region above the x -axis bounded by these curves.
- B) Set up, but Do Not Evaluate the definite integral(s) which describe the shaded region in part A.

IV

Let $F(t)$, $G(t)$, and $H(t)$ be functions such that: $G(t) = F'(t)$ and $H(t) = G'(t)$ on $[0, 1]$. (15 pts total)Suppose $F(0) = 0$, $F(1) = 1$, $G(0) = 0$, $G(1) = 0$, $H(0) = 2$, and $H(1) = 1$

Find and briefly explain your reasoning:

a) $\int_0^1 G(t) dt$ b) $\int_0^1 H(t) dt$

V

Consider the area under the curve $f(x) = x^2$ over the closed interval $[0, 2]$ (25 pts tot)

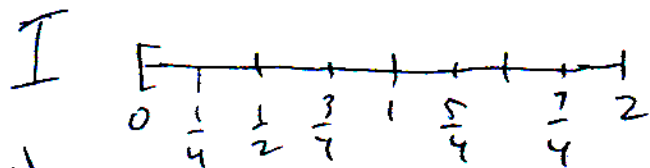
- a) Draw a clear, labeled, easy to read graph of the situation.
- b) Partition $[0, 2]$ into 4 sections of equal length. List the partition points.
- c) What is the mesh of the partition?
- d) Choose sampling numbers to be the midpoints of each section. List the sampling numbers.
- e) Write an approximating sum using these partition points and sampling numbers. Do not evaluate.
- f) Rewrite part e above using sigma notation.

Extra Credit 5 pts

Consider the following definite integral and its solution: $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{3} \Big|_{-1}^2 = -\frac{3}{8}$

"Since the function $f(x) = x^{-4}$ is positive, the area under it and above the x -axis is positive. The integral should also be positive."

Agree or disagree with the above quote and defend your position.



a)

1) x_i is the sample point of the section.

$$x_i \in \left\{ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4} \right\}$$

2) $f(x_i)$ is the length of the cross sections at each sample point.

$$f(x_i) \in \left\{ \frac{3}{8}, \frac{27}{8}, \frac{75}{8}, \frac{147}{8} \right\}$$

3) $f(x_i)(x_i - x_{i-1})$ is the area of approx rectangle. $\frac{1}{2}$ each time

$$\in \left\{ \frac{3}{16}, \frac{27}{16}, \frac{75}{16}, \frac{147}{16} \right\}$$

4) $(x_i - x_{i-1})$ is the mesh (if it is the width of the sections, in this problem all sections are $\frac{1}{2}$ wide).

1)

$$\text{II } f'(x) = g \quad f'' = -f \quad h = f^2 + g^2$$

$$h_1 = f^2 + (f')^2$$

$$\begin{aligned} h'(x) &= 2f f'(x) + 2f' f'' \\ &= 2fg + 2g(-f) \\ &= 2fg - 2fg \\ &= 0 \end{aligned}$$

so $h'(x) = 0$ ie h is a constant fcn.

Since $h(0) = 5$ then $h(10) = 5$ also

$$6 \cdot \frac{1}{16}$$

$$\frac{3}{8}$$

$$3 \cdot 25$$

$$49$$

$$\frac{3 \cdot 9}{16}$$

$$B) \frac{3}{16} + \frac{27}{16} + \frac{75}{16} + \frac{147}{16}$$

$$\frac{3}{16} [1 + 9 + 25 + 49]$$

$$C) \frac{3}{16} \sum_{i=1}^4 (2i-1)^2$$

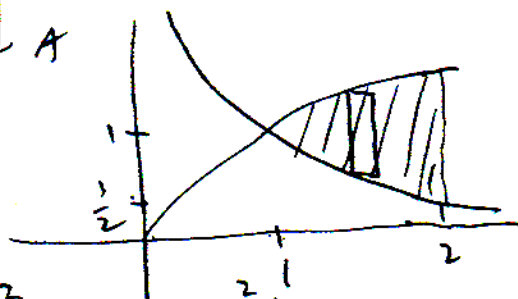
$$D) \int_0^2 6t^2 dt =$$

$$\frac{6(2)^3}{3} - \frac{6(0)^3}{3}$$

$$2 \cdot 8 - 0$$

$\frac{16}{16}$
The object travels 16 feet.

III A



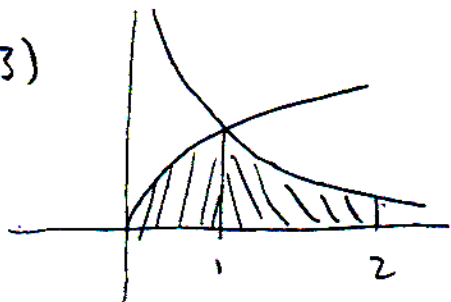
$$\int_1^2 \sqrt{x} \cdot \frac{1}{x} dx = \int_1^2 \frac{1}{\sqrt{x}} dx$$

$$\frac{2}{3} x^{3/2} \Big|_1^2 - \ln x \Big|_1^2$$

$$\frac{2\sqrt{8}}{3} - \frac{2}{3} - \ln 2 + \ln 1$$

$$\frac{4\sqrt{2}-2}{3} - \ln 2$$

III B)

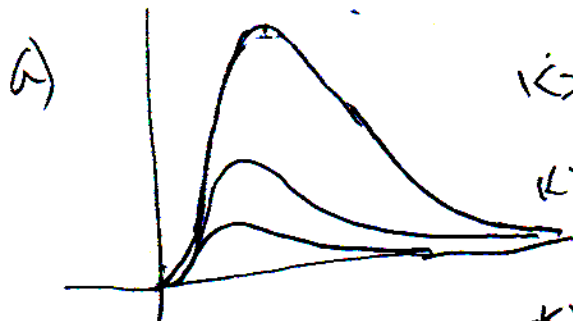


$$\int_0^1 x^{\frac{1}{2}} dx + \int_1^2 \frac{1}{x} dx$$

$$\frac{2}{3} x^{\frac{3}{2}} \Big|_0^1 + \ln x \Big|_1^2 = \frac{2}{3} - 0 + \ln 2 - 0 = \frac{2 + 3 \ln 2}{3}$$

IV $f(x) = 10 x e^{-kx}$

$k=1 (1, 3.679) \quad \left(\frac{1}{k}, \frac{10}{k} e^{-1}\right)$



$k=2 (.5, 1.839)$

$k=3 (.333, 1.228)$

c) as k gets bigger the max pt gets closer to the origin.

d) the max value lies on the y where $\frac{10}{e} x$

b) $f'(x) = e^{-kx} + K x e^{-kx}$
 $e^{-kx} [1 - kx] = 0$ set
 $x = \frac{1}{k}$

$f\left(\frac{1}{k}\right) = 10 \cdot \frac{1}{k} e^{-k\left(\frac{1}{k}\right)}$
 $10 \frac{1}{k} e^{-1}$

k	max	$\frac{10}{e}$
1	$\frac{10}{e}$	Slope is $\frac{10}{e}$
2	$\frac{10}{2e}$	$\lim_{n \rightarrow \infty} \frac{10}{ne} \rightarrow 0$
3	$\frac{10}{3e}$	So y intercept is 0
$\vdots$	$\vdots$	$y = \frac{10}{e} x$
N	$\frac{10}{Ne}$	

EXTRA

$f(x) = \int_0^x \sin^{-1} t dt = \int_0^x \sin^{-1} t dt$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$f'(x) = \sin^{-1} u \cdot du$

$\sin^{-1} \sin x \cos x$

$\sin^{-1}(\sin(\pi-x)) \cos x$

$f'(x) = (\pi-x) \cos x$

But $\frac{\pi}{2} < x < \pi$
 since we are in 2nd quadrant,
 $\sin^{-1}(\sin x) = \sin^{-1}(\sin(\pi-x))$
 so

1. 1000 1000
 1000

1001 2
 201 4
 801 9
 701 4
 601 1
 570 0

$$\frac{10}{2e} - \frac{10}{e}$$

2-1



85.05 are 62

$$\begin{array}{r} 3.679 \\ 1.837 \\ \hline (1.842) 2 \\ 3.684 \end{array}$$

Advanced Placement Calculus

Name Calc Ch 1-8

Copy original problem.

Per _____

Date 9-7Convince *me* that you understand the concept!

Chapters 1 – 8 Exam

I An object travels at a speed of $6t^2$ feet per second, t seconds after it starts to move. Set up an approximating sum for the distance traveled by this object in the first two seconds by partitioning the interval $[a, b]$ into four equal sections. The sample points will be the midpoints of the sections. (No calculators in this section.) (40 pts tot)

a) Explain in general terms what each of the following items is *and* its numeric value(s) as used in this problem.

1) X_i 2) $f(X_i)$ 3) $f(X_i)(x_i - x_{i-1})$ 4) $(x_i - x_{i-1})$

b) Express the approximation in expanded form of the sum.

c) Express the approximation in a simplified summation notation.

d) Using the first Fundamental Thm of Calculus, compute the exact distance traveled by this object in the first two second of movement. (Be sure to show that you know what the 1st FTC is.)

II Suppose f, g, h are all differentiable functions. ($f^2(x)$ means $(f(x))$ times $(f(x))$) (20 pts)
Given Show that if $f'(x) = g(x)$; $f''(x) = -f(x)$; $h(x) = f^2(x) + g^2(x)$;
and $h(0) = 5$. Find $h(10)$.

III Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$. Draw the graph showing the area to be computed and compute the exact area described (No calculators.) (10 pts ea)

a) $\int_1^2 f(x) dx - \int_1^2 g(x) dx$ b) $\int_0^1 f(x) dx + \int_1^2 g(x) dx$

IV Consider the function: $f(x) = 10xe^{-kx}$ where k is a positive constant. (20 pts tot)

a) Use graphing window $0 \leq x \leq 5$, $0 \leq y \leq 5$.

Sketch the graphs of f for these three values: $k \in \{1, 2, 3\}$.

b) Find the x and y coordinates of the maximum points for each graph. Express the coordinates for the maximum of f in terms of k .

c) Use your answer to part b above to explain how the three graphs drawn in part (a) are related to each other.

d) Prove or explain that the maximum points of f for different values of k all lie on the same straight line. State the equation of the line.

Extra Credit ----- 5 pts -----

Find $f'(x)$ if $f(x) = \int_0^{\sin x} \sin^{-1} t \, dt$ and $\frac{\pi}{2} < x < \pi$.

- I
- x_i The sample point which in the section $x_i - x_{i-1}$, this is the x value the function will act upon.
 - $x_i - x_{i-1}$ width of the i^{th} section. This is the width of the approximating rectangle. It is the operational part of telescoping sums.
 - $f(x_i)$ height of the i^{th} approximating rectangle. Used to find the area of the approximating rectangle.
 - $f(x_i)(x_i - x_{i-1})$ The area of the i^{th} approx rectangle. These are summed to approximate the area under the curve. When the limit is applied to this sum, the area becomes exact.
 - nish is the width of the widest section. As the nish goes to zero the section $x_i - x_{i-1}$ also shrinks toward zero. The final result is the exact area under the curve.

II

$$\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots + \frac{1}{100}$$

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{50} \right)$$

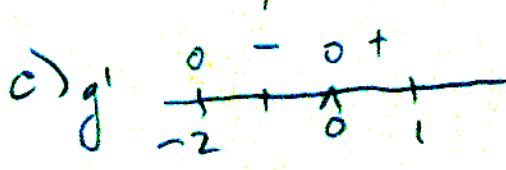
$$\frac{1}{2} \sum_{i=2}^{50} \frac{1}{i}$$

III

$y = f(x)$
 $g(x) = \int_1^x f(t) dt$
 (B) $g'(x) = f(2x) \cdot 2 \stackrel{\text{Set}}{=} 1$
 $f(2x) = \frac{1}{2}$
 $2x \in \frac{1}{2}$ (from graph)
 $x = \frac{1}{4}$

a) $-4 \leq t \leq 2$
 $-4 \leq 2x \leq 2$
 $-2 \leq x \leq 1$

$\frac{5}{2} \leq t \leq \frac{9}{2}$
 $\frac{5}{2} \leq 2x \leq \frac{9}{2}$
 $\frac{5}{4} \leq x \leq \frac{9}{4}$
 Does not include $t = -1$
 $\therefore$ not part of Domain by FTC, fct must be continuous



$g'(x) = 2f(2x) \stackrel{\text{Set}}{=} 0$
 $f(2x) = 0$
 $2x = 0 \quad 2x = -4$
 $x = 0 \quad x = -2$

$g(-2) = \int_{-2}^{-1} f(t) dt = - \int_{-4}^{-1} f(t) dt = -(-2.5) = 2.5$
 $g(0) = \int_{-1}^0 f(t) dt = -\frac{1}{2}$
 $g(-\frac{1}{2}) = \int_{-1}^{-\frac{1}{2}} f(t) dt = 0$

$g(\frac{1}{2}) = \int_{-1}^{\frac{1}{2}} f(t) dt = 0$
 $g(1) = \int_{-1}^1 f(t) dt = -\frac{1}{2} + 2 = \frac{3}{2}$

max $(-2, 2.5)$ End of dec limit.
 min $(0, -\frac{1}{2})$ f' test. $\therefore$ interval
 max $(1, \frac{3}{2})$ $\leftarrow$ would inc to it.
 $(-\frac{1}{2}, 0)$ $\leftarrow$ interval $(\frac{1}{2}, 0)$



IV a) $d\left(\int_0^x \sin t^2 dt\right)$

$$\frac{d(\sin x^2)}{2x \cos x^2} \bigg|_{x=3}$$

$$6 \cos 9$$

c) $\int_1^2 \ln(1+t^2) dt$

$$d(\ln(5))$$

d) $d\left(\int_1^x \frac{1}{t} dt\right)$

$$d\left(\frac{1}{x}\right)$$

$$-\frac{1}{x^2} \bigg|_{x=3} = -\frac{1}{9}$$

b) $\int_0^{\sin x} \sin^{-1} t dt$

$$\sin^{-1}(\sin x) \cos x$$

Since $x=3$, $\sin x = \sin(\pi-x)$

$$\sin^{-1} \sin(\pi-x) \cos x$$

$$d((\pi-x) \cos x)$$

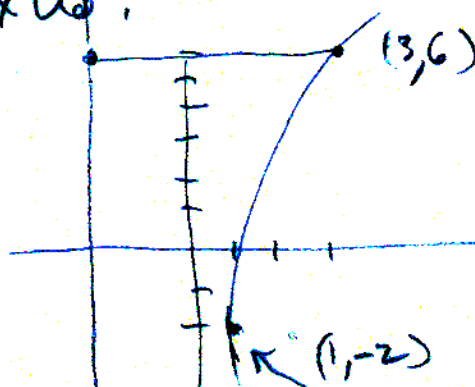
$$-\cos x + (\pi-x)(-\sin x) \bigg|_{x=3}$$

$$-\cos 3 - (\pi-3) \sin 3$$

V $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{3} \bigg|_{-1}^2 = -\frac{3}{8}$

Dis + free. The fctn x^{-4} is not continuous through the interval (specifically at $x=0$). The $-\frac{3}{8}$ has nothing to do with anything including area.

Ex 10.



$$x = \frac{1}{4c}(y-k)^2 + h$$

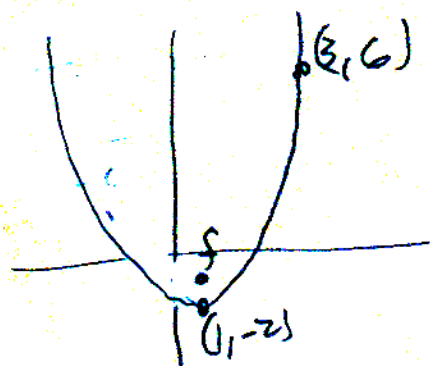
$$x-1 = \frac{1}{4c}(y+2)^2 + 1$$

(3, 6)

$$2 = \frac{1}{4c}(6+2)^2$$

$$c = 8$$

focus (8, -2)



$$y = \frac{1}{4c}(x-h)^2 + k$$

$$6 = \frac{1}{4c}(3-1)^2 - 2$$

$$8 = \frac{1}{4c}(4) \therefore c = \frac{1}{8}$$

focus $(1, -\frac{15}{8})$

2/19/98

100	0	93
90	2	
80	4	
70	3	
60	5	
50	2	56

$$\frac{1195}{16} = 74.7$$

a few cosmetic
errors as marked.
change the quote
as indicated.

Advanced Placement Calculus

Copy original problem.

Convince *me* that *you* understand the concept!

No Calculators.

Name

Per

Date

Chapters 1 - 8 Exam

I

The definite integral of a function f over an interval $[a, b]$ is denoted by $\int_a^b f(x) dx$ and is defined as follows: $\int_a^b f(x) dx = \lim_{\text{mesh} \rightarrow 0} \sum_{i=1}^n f(X_i)(x_i - x_{i-1})$. ~~Describe each of the following items and describe its purpose in the context of our class.~~ *Keep* (4 pts ea)

~~a) Explain in general terms what each of the following items is and its numeric value(s) as used in this problem.~~

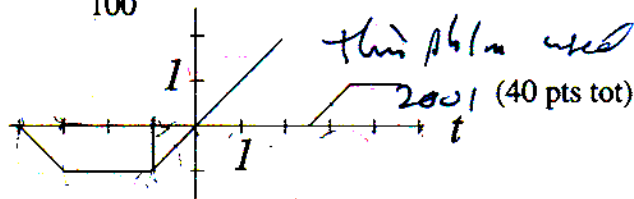
- 1) X_i 2) $(x_i - x_{i-1})$ 3) $f(X_i)$ 4) $f(X_i)(x_i - x_{i-1})$ 5) mesh

II

Write the following in summation notation: $\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots + \frac{1}{100}$ (5 pts)

III

Let $g(x) = \int_{-1}^{2x} f(t) dt$ when given the graph of $y = f(t)$:



- A) Explain clearly why the domain is: $D_g: -2 \leq x \leq 1$. Include reasoning how you know there is not a second interval.
B) For what values of x does $g'(x) = 1$?
C) Determine the coordinates of all extrema and all axis intercepts.
D) Draw $g(x)$.

IV

Find the second derivative with respect to x evaluated at $x = 3$ for each of the following: (25 pts tot)

- a) $\int_0^x \sin t^2 dt$ b) $\int_0^{\sin x} \sin^{-1} t dt$ c) $\int_1^2 \ln(1+t^2) dt$ d) $\int_t^x \frac{1}{t} dt$

V

Consider the following definite integral and its solution: $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{3} \Big|_{-1}^2 = -\frac{3}{8}$ (10 pts)

"there is such a thing as negative area and this is an example"
~~"Since the function $f(x) = \frac{1}{x^4}$ is positive for all x , the area under it and above the x -axis should also be positive."~~
ab it.

Agree or disagree with the above quote and defend your position.

Extra Credit ----- 5 pts -----

Find the coordinates of the focus of the parabola which has vertex at $(1, -2)$ and passes through the point $(3, 6)$.

$$I \int_2^4 6 dx = 12$$

$$W 2 = x_0 < x_1 < x_2 \dots < x_n = 4$$

$$\sum_{i=1}^n \bar{x}_i (x_i - x_{i-1})$$

use of
FTC
= -4

$$\sum_{i=1}^n 6 (x_i - x_{i-1})$$

$$6(4-2)$$

$$6 \cdot 2$$

$$12$$

$$(0, 2)$$

$$y = x + 2$$

$$y = x$$

$$(\sqrt{2}, \sqrt{2})$$

$$(-2, 0)$$

III

$$x^2 + x^2 = 4$$

$$2x^2 = 4$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

"this y value minus this x value"

V

$$\int_0^{50} v(t) dt$$

$$IV \frac{1}{b-a} \int_a^b f(x) dx = f_{ave}$$

$$a) x \in [0, \sqrt{\pi}] \quad y \in [0, 1]$$



$$c) \text{ about } \frac{1}{2}$$

$$D) \frac{.8947}{\sqrt{\pi}} = .5048$$

$$E) \frac{.894831}{\sqrt{\pi}} = .5049$$

F) The exact value is slightly larger than approx since $N=100$ is approx and $N \rightarrow \infty$ is integral.

$$g(x) = \int_x^2 \sqrt{\cos t} dt + \int_2^3 \sqrt{\cos t} dt$$

$$g(x) = \int_x^2 \sqrt{\cos t} dt = \text{constant}$$

$$g'(x) =$$

$$= -2 \times \sqrt{\cos x} \times 1$$

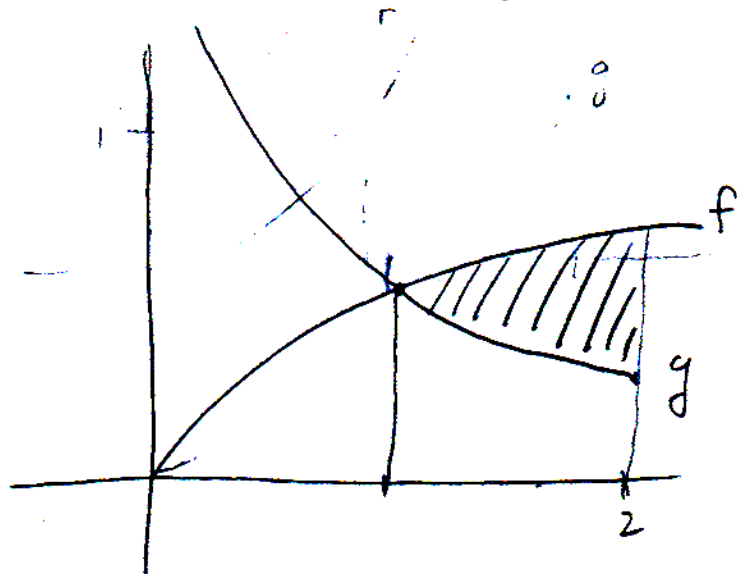
$$\int_{-2}^0 x+2 dx + \int_0^{\sqrt{2}} \sqrt{4-x^2} - y dy$$

$$10(12) + 10(30) + 10(70) + 10(81) + 10(60)$$

$$\begin{array}{r} 120 \\ 300 \\ 700 \\ 810 \\ 600 \\ \hline 2530 \end{array}$$

This integral is the total distance traveled in feet during the time from zero to 50 seconds.

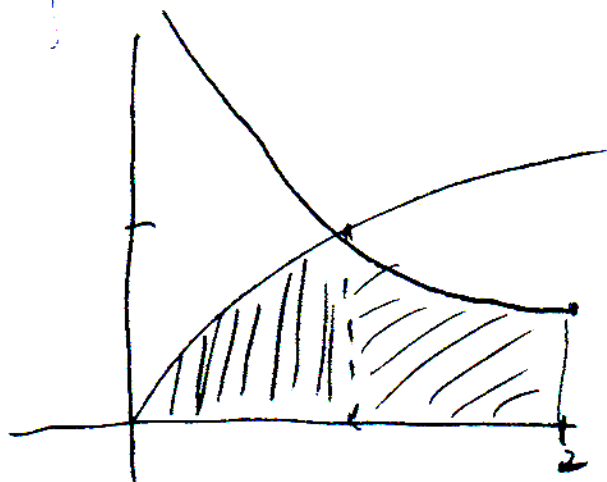
VT $f(x) = \sqrt{x}$ $g(x) = \frac{1}{x}$



$$\int_1^2 \sqrt{x} dx - \int_1^2 \frac{1}{x} dx$$

$$\frac{2}{3} x^{\frac{3}{2}} \Big|_1^2 - (\ln 2 - \ln 1)$$

$$\frac{4\sqrt{2}}{3} - \frac{2}{3} - \ln 2 = .526$$



$$\int_0^2 \sqrt{x} dx + \int_1^2 \frac{1}{x} dx$$

$$\frac{2}{3} x^{\frac{3}{2}} \Big|_0^2 + \ln 2$$

$$\frac{4\sqrt{2}}{3} + \ln 2 = 1.36$$

EX 7.14

$$f(x) (b-a) = \int_a^b f(x) dx$$

$$f(x) (b-a) = \int_a^b \sin^2 x^2 dx$$

$$f(x) = \frac{1}{\sqrt{\pi}} \quad (.89485...)$$

$$\sin^2 x^2 = .50485$$

$$x^2 = .52921$$

$$x = .7275$$

100	0
90	4
80	3
70	3
60	2
50	1

97

54

$$\begin{array}{r} 1030 \\ 13 \end{array}$$

79.2

Question

I had an error on the

$\cos 2 < 0$

$\cos 3 < 0$

It should have been

something like

$$\frac{1}{x^2} \int_{.73}^{1.4} \sqrt{\cos t} dt + \int_{.73}^{1.4} \sqrt{\cos t} dt$$

Chapters 7 & 8 Exam

I

Use the ~~definition of definite integral~~ to show that $\int_2^6 dx = 12$.

(hint: The definition has nothing to do with a graph.)

II

Differentiate: $g(x) = \int_{x^2}^2 \sqrt{\cos t} dt + \int_2^3 \sqrt{\cos t} dt$

$$g(x) = \int_{x^2}^2 \sqrt{\cos t} dt + \int_2^3 \sqrt{\cos t} dt$$

III

Given the three equations: $x^2 + y^2 = 4$, $y = x + 2$, and $y = x$.

(20 pts total)

A) Sketch the three equations on the same axis. Shade the region above the x-axis bounded by these curves.

B) Set up, but Do Not Evaluate the definite integral(s) which describe the shaded region in part A. Give a very explicit explanation of your methods.

IV

The value $\frac{1}{b-a} \int_a^b f(x) dx$ is called the mean (or average) value of f on the interval $[a, b]$ and is usually denoted f_{av} . Let $f(x) = \sin x^2$ for $0 \leq x \leq \sqrt{\pi}$

(20 pts total)

- Describe the window setting necessary to yield a clear view of the function in the given interval on your calculator.
- Sketch your graph.
- Predict the mean value, f_{av} , of f on $[0, \sqrt{\pi}]$. Call your prediction A.
- Use the program RiemannC with $n = 100$ to find the approximation of the integral. Compute f_{av} using this value and call this value B.
- Use fnInt ($Y_1, X, 0, \sqrt{\pi}$) / $\sqrt{\pi}$ (found in the MATH menu). Call this value C.
- Compare the values A, B, and C. Briefly explain.

V

The velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown in a table of values for $v(t)$, at 5 second intervals of time.

Approximate the definite integral, $\int_0^{50} v(t) dt$ with a riemann sum, using the mid-

points of five subintervals of equal length. Using correct units, explain the meaning of this integral. (20 pts)

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

VI

Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$. Draw the graph showing the area computed and compute the exact area described. (tot 20)

$$a) \int_1^2 f(x) dx - \int_1^2 g(x) dx$$

$$b) \int_0^1 f(x) dx + \int_1^2 g(x) dx$$

Extra Credit 5 pts

Lemma 3, used by our text in the proof of the Fundamental Theorem of Calculus states:

$$f(X)(b-a) = \int_a^b f(x) dx. \text{ Using the function in Section IV. Compute the "cap X" value.}$$

Calculator 101

- Let f be the function given by $f(x) = \cos x$. If the number c satisfies the conclusion of the Mean Value Theorem (also known as the Law of the Mean) for f on the closed interval $[2, 4]$, then $c =$
a) 0.119 b) 2.902 c) 3.023 d) 3.142 e) 3.261
- The rate of decay of a radio active substance is proportional to the amount of the substance present at any time t . In 1840 there were 50 grams of the substance and in 1910 there were 35 grams. To the nearest gram, how many grams of the substance remained in 1990?
a) 18 b) 20 c) 23 d) 36 e) 107
- If $f(x) = \sqrt[5]{x^3 - 2x}$, then $f'(\sqrt{3}) =$
a) 0.129 b) 0.139 c) 0.902 d) 0.906 e) 1.116
- The slope of the line containing the point $(4, 0)$ and tangent to the graph of $y = e^{-x}$ is
a) 0.050 b) 0.018 c) -0.050 d) -0.018 e) -0.070
- If $f(x) = \ln x$, then the average rate of change of f on the interval $[2, 5]$ is
a) 0.100 b) 0.133 c) 0.305 d) 1.155 e) 1.220
- A 16-foot ladder is leaning against a vertical wall. The top end of the ladder is sliding down the wall at the rate of 5.6 feet per second. What is the rate of change, in radians per second, of the angle θ (the angle opposite the vertical wall) at the instant when the top of the ladder is 7 feet above the ground?
a) -6.2 b) -0.8 c) -0.4 d) -0.3 e) -0.1
- The slope of the line tangent to the curve $y = x^{-x}$ at $x = 0.368$ is closest to the integer
a) -6 b) -1 c) 0 d) 1 e) 6

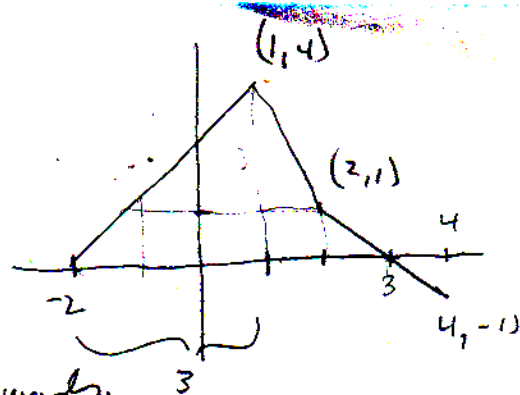
8. Let $f(x) = \sin x$ and $g(x) = x^2$. Graph f and g . Which of the following points best approximates the intersection of the two graphs for $x > 0$? a) $(2, 1)$ b) $(.9, .9)$ c) $(4, -1)$
9. Find all solutions of $\sqrt{x^3} + x \leq -1$ by using a graph.
10. Describe or sketch the graph of $f(x) = \ln\left(x + \sqrt{x^2 - 4}\right) + \ln\left(x - \sqrt{x^2 - 4}\right)$.
11. Graph $f(x) = 2x^{1/2} - x^2$. Which of the following appears to be nearest any zeros of f ?
a) -2 b) 2 c) $.5$ d) 1.5
12. Let $f(x) = \frac{1}{1 + \cos x^{-2}}$. Approximate $f(1.1)$ to 4 decimal places.
13. Let $f(x) = 3x^{1/2} + x^{2/3}$. Approximate $f(1.3)$ to 4 decimal places.
14. Find all solutions of $x^4 + 3x + 5 > 0$. Express your answer as an interval or union of intervals.
15. Graph $f(x) = 1 + \sqrt{x}$, $g(x) = 1 + \sqrt{1 - x}$ and $h(x) = (f + g)(x)$. ie $f(x) + g(x)$ and $k(x) = f(g(x))$. ie $f \circ g$. Describe the graphs of h and k .
16. Graph $f(x) = x^5 + 4x^2 + x$ using a range of $[-4, 4]$ on both axes. Which, if any, of the following appear to be nearest the zeros of f ?
a) -2 b) -1.5 c) 0 d) 1
17. Describe and sketch the graph of $f(x) = \ln(3x) - \ln x$.
18. Two particles start at the origin and move along the x -axis. For $0 \leq t \leq 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?
a) none b) one c) two d) three e) four
19. If c satisfies the conclusion of the Mean Value Theorem for $f(x) = x^4 - 3x^3 + 2x - 3$ on the interval $-1 \leq x \leq 2$, then c could be
a) -0.59 only b) 0.00 only c) 0.84 only
d) -0.59 and 0.84 e) -0.43 and 0.54
20. A car with constant acceleration goes from 0 to 88 feet per second in 440 feet. How long does it take?

I

a) $g(4) = \int_1^4 f(t) dt$

$\frac{1}{2} \cdot 3 \cdot 1 + 1 + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{2}$

$g(-2) = -\frac{1}{2} \cdot 3 \cdot 4 = -6$ ^{2pts} *why because $\int_{-2}^1 f(t) dt$ backwards.*



b) $g'(1) \Rightarrow f(1) = 4$ from graph.
because $g'(x) = f(x)$ by FTC.

c) g' $\begin{array}{c} + \quad 0 \quad - \\ \hline -2 \quad 3 \quad 4 \end{array}$ $(2, -6)$ is a min in g' ,
 $(3, 3)$ is a max g' test.

$g(3) = \int_1^3 f(t) dt = \frac{1}{2} \cdot 3 \cdot 1 + 1 + \frac{1}{2} = 3$

$(4, \frac{5}{2})$ is a min in g test.

d) $g'' = f'$ $\begin{array}{c} + \quad 0 \quad - \\ \hline -2 \quad 1 \quad 2 \quad 3 \quad 4 \end{array}$ The 2nd der dNE when the slope of f DNE and this happens when slope doesn't become smooth change in "sharp corners".

e) $x=1$ is a pI because

g changes from concave up to down.

$x=2$ is not a pI because g is concave down on both sides.

Neither 2 or 6

II

- a) $\begin{cases} 0 \\ 3 \\ 6 \\ 9 \\ 12 \\ 15 \\ 18 \\ 21 \\ 24 \end{cases}$

width = 6

$6(10.4 + 11.2 + 11.3 + 10.2)$

$6(43.1)$

258.6

In 24 hours approximately 258.6 gallons of water flowed through the pipe.

IV a) $d\left(\int_0^x \sin t^2 dt\right)$

$$\frac{d(\sin x^2)}{2x \cos x^2} \bigg|_{x=3}$$

$$6 \cos 9$$

c) $\int_1^2 \ln(1+t^2) dt$

$$d(\ln(5))$$

d) $d\left(\int_1^x \frac{1}{t} dt\right)$

$$d\left(\frac{1}{x}\right)$$

$$-\frac{1}{x^2} \bigg|_{x=3} = -\frac{1}{9}$$

b) $\int_0^{\sin x} \sin^{-1} t dt$

$$\sin^{-1}(\sin x) \cos x$$

Since $x=3$, $\sin x = \sin(\pi-x)$

$$\sin^{-1} \sin(\pi-x) \cos x$$

$$d((\pi-x) \cos x)$$

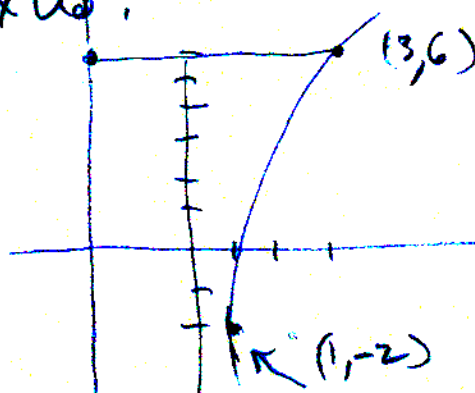
$$-\cos x + (\pi-x)(-\sin x) \bigg|_{x=3}$$

$$-\cos 3 - (\pi-3) \sin 3$$

V $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{3} \bigg|_{-1}^2 = -\frac{3}{8}$

Disagree. The fctn x^{-4} is not continuous through the interval (specifically at $x=0$). The $-\frac{3}{8}$ has nothing to do with anything including area.

Ex: 10



$$x = \frac{1}{4c}(y-k)^2 + h$$

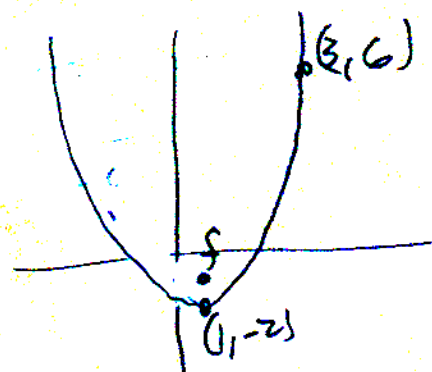
$$x-1 = \frac{1}{4c}(y+2)^2 + 1$$

(3, 6)

$$2 = \frac{1}{4c}(6)^2$$

$$c = 8$$

focus (8, -2)



$$y = \frac{1}{4c}(x-h)^2 + k$$

$$6 = \frac{1}{4c}(3-1)^2 - 2$$

$$8 = \frac{1}{4c}(4) \therefore c = \frac{1}{8}$$

focus $(1, -\frac{15}{8})$

100 100

100	1
90	6
80	6
70	5
60	
50	1

157

$$\frac{1586}{19} = 83.5$$

Greater than.

Copy the original problem.

Convince me that you understand the concept.

Per

Date

CHAPTER #7 & 8

I

The definite integral of a function f over an interval $[a, b]$ is denoted by $\int_a^b f(x) dx$ and definedas follows: $\int_a^b f(x) dx = \lim_{\text{mesh} \rightarrow 0} \sum_{i=1}^n f(X_i)(x_i - x_{i-1})$. Describe each of the following and its purpose? (25 pts tot)

- a) X_i b) $x_i - x_{i-1}$ c) $f(X_i)$ d) $f(X_i)(x_i - x_{i-1})$ e) mesh

II

A rocket travels at a speed of $6t^2$ feet per second t seconds after its projection.

- a) Obtain an approximating sum for the distance traveled by this rocket in the first two seconds, using the partition of the interval $[a, b]$ into four equal sections and the midpoint of each section as its sampling point.
- Express the result in expanded form of the sum. (5 pts)
 - Specifically state the value of $f(X_i)$. Specifically state the value of $(x_i - x_{i-1})$. (10 pts)
 - Express the result in a simplified summation notation. (5 pts)
- b) Using the first Fundamental Theorem of Calculus, compute the exact distance traveled by this rocket in the first two seconds. (10 pts)

III

Suppose f is a differentiable function. Show that if $x \neq 0$, that any maximum or minimum of $g(x) = xf(x) - \int_0^x f(t) dt$ occurs when $f'(x) = 0$. Explain. (20 pts)

IV

Let f be a function that is even and continuous over $[-3, 3]$. The function and its derivatives have the properties indicated in the table:

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

- a) Find the x -coordinate of any max or min. Identify. Justify of course.
- b) Find x -coordinate of any P.I.

c) Sketch.

(25 pts total)

EXTRA CREDIT

5 pts

V

Find $f'(x)$ if $f(x) = \int_0^{\sin x} \sin^{-1} t dt$ $\frac{\pi}{4} < x < \pi$.

100	2
90	9
80	7
70	0
60	2
50	1

$$\frac{1855}{21} = 88.3$$

without the 50
90.3

102

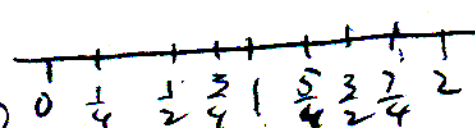
50

This was 1 question
Short of what it
Should have been
make part I worth 10pts
and ~~other~~ ^{new section worth} 15pts can be
a graph and shows
cross sections type.

10pts
15pts

- I
- Δx_i sample point in section $[x_{i-1}, x_i]$. Depending on the Δx_i , this changes the approx from over to under or vice versa
 - $x_i - x_{i-1}$ width of a section. The largest section is called the mesh. As the mesh $\rightarrow 0$, this section decreases in width to yield the area.
 - $f(x_i)$ height of current rectangle, area is height times width at the particular section.
 - $f(x_i)(x_i - x_{i-1})$ area of the i^{th} rectangle. These are summed to approx the area under the curve
 - mesh The largest of all sections in the interval. as the mesh shrinks to zero all sections also shrink to zero ending up by summing line lengths to get area.

II

a) 

① $6 \left[\frac{1}{2} \left(\frac{1}{4} \right)^2 + \frac{1}{2} \left(\frac{3}{4} \right)^2 + \frac{1}{2} \left(\frac{5}{4} \right)^2 + \frac{1}{2} \left(\frac{7}{4} \right)^2 \right]$
 $3 \left[\left(\frac{1}{4} \right)^2 + \dots \right]$
 $\frac{3}{16} [1 + 3^2 + 5^2 + 7^2]$

② $f(x_i) = \left(\frac{2x_i - 1}{4} \right)^2$
 $x_i - x_{i-1} = \frac{1}{2}$

③ $\frac{3}{16} \sum_{i=1}^4 (2i-1)^2$

b) $\int_0^2 6t^2 dt = \frac{2(2)^3 - 2(0)^3}{2 \cdot 3} = \frac{16}{3}$
 16 feet

III

$$g(x) = x f(x) - \int_0^x f(t) dt$$

$$g'(x) = f(x) - x f'(x) - f(x)$$

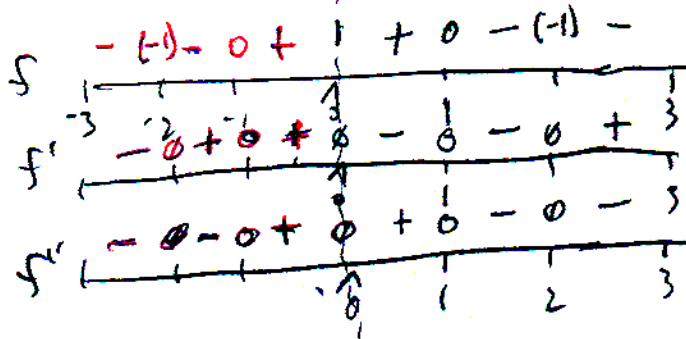
$$= -x f'(x) \stackrel{?}{=} 0$$

$x \neq 0 \therefore f'(x) \stackrel{?}{=} 0$

$$\begin{array}{c} + \quad - \\ - \quad 0 \quad + \\ \hline 0 \quad \quad x \end{array}$$

If $x > 0$, max, if $x < 0$, min
 Thus the extremum occurs when
 $f'(x) = 0$

IV



f is even $\therefore f'$ odd $\neq f''$ even
reflects at $x=0$

$$-f(x) = f(-x)$$

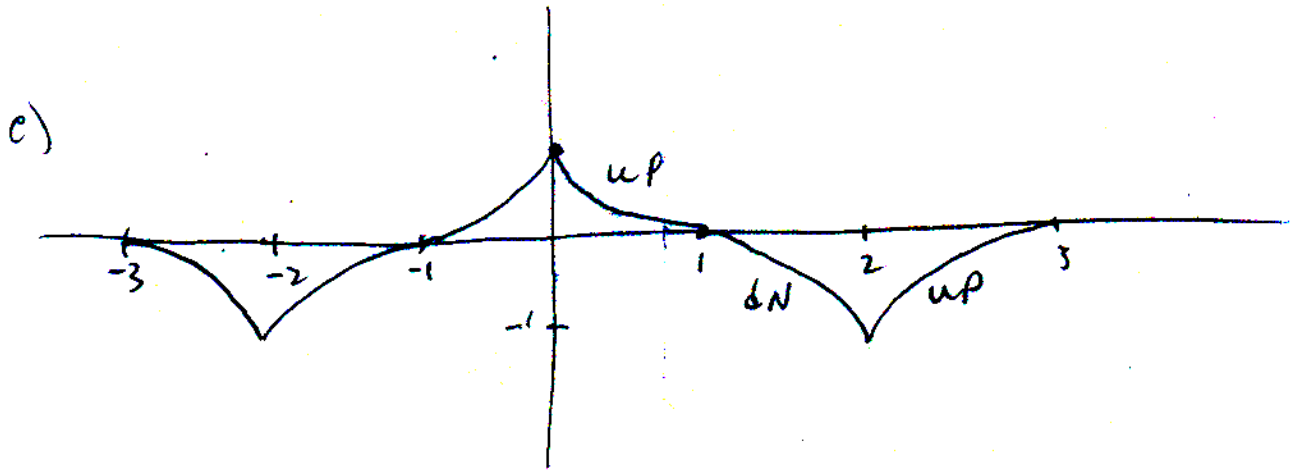
B) P.I.

$$x = -1$$

$$x = 1$$

f'' zero and
 f'' change signs

- a)
- $x = -3$ max dec then inc
 - $x = -2$ min dec then inc
 - $x = 0$ max inc then dec
 - $x = 2$ min dec then inc
 - $x = 3$ max inc to it.



Extra.

$$f(x) = \int_0^{\sin x} \sin^{-1} t \, dt \quad \frac{\pi}{2} < x < \pi$$

$$\begin{aligned} \sin x &= \sin(\pi - x) \\ &= \sin \pi \cos x - \sin x \cos \pi \\ &= \sin x \end{aligned}$$

$$f(x) = \int_0^{\sin(\pi-x)} \sin^{-1} t \, dt$$

$$f'(x) = \sin^{-1}(\sin(\pi-x)) (\cos(\pi-x)(-1))$$

$(\pi-x) \cos x.$

$$\begin{aligned} &\cos(\pi-x) \\ &= (\cos \pi \cos x + 0)(-1) \\ &= \cos x \end{aligned}$$

change sign

Advanced Placement Calculus

Name Calc Ch 1-8

Copy original problem.

Per _____

Date 9-7Convince *me* that you understand the concept!

Chapters 1 – 8 Exam

I An object travels at a speed of $6t^2$ feet per second, t seconds after it starts to move. Set up an approximating sum for the distance traveled by this object in the first two seconds by partitioning the interval $[a, b]$ into four equal sections. The sample points will be the midpoints of the sections. (No calculators in this section.) (40 pts tot)

a) Explain in general terms what each of the following items is *and* its numeric value(s) as used in this problem.

1) X_i 2) $f(X_i)$ 3) $f(X_i)(x_i - x_{i-1})$ 4) $(x_i - x_{i-1})$

b) Express the approximation in expanded form of the sum.

c) Express the approximation in a simplified summation notation.

d) Using the first Fundamental Thm of Calculus, compute the exact distance traveled by this object in the first two second of movement. (Be sure to show that you know what the 1st FTC is.)

II Suppose f, g, h are all differentiable functions. ($f^2(x)$ means $(f(x))$ times $(f(x))$) (20 pts)

Given Show that if $f'(x) = g(x)$; $f''(x) = -f(x)$; $h(x) = f^2(x) + g^2(x)$; and $h(0) = 5$; Find $h(10)$.

III Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$. Draw the graph showing the area to be computed and compute the exact area described (No calculators.) (10 pts ea)

a) $\int_1^2 f(x) dx - \int_1^2 g(x) dx$ b) $\int_0^1 f(x) dx + \int_1^2 g(x) dx$

IV Consider the function: $f(x) = 10xe^{-kx}$ where k is a positive constant. (20 pts tot)

a) Use graphing window $0 \leq x \leq 5$, $0 \leq y \leq 5$.

Sketch the graphs of f for these three values: $k \in \{1, 2, 3\}$.

b) Find the x and y coordinates of the maximum points for each graph. Express the coordinates for the maximum of f in terms of k .

c) Use your answer to part b above to explain how the three graphs drawn in part (a) are related to each other.

d) Prove or explain that the maximum points of f for different values of k all lie on the same straight line. State the equation of the line.

Extra Credit ----- 5 pts -----

Find $f'(x)$ if $f(x) = \int_0^{\sin x} \sin^{-1} t \, dt$ and $\frac{\pi}{2} < x < \pi$.

1. 1000 1000
 1000

1000 2
 200 4
 800 9
 200 4
 600 1
 500 0

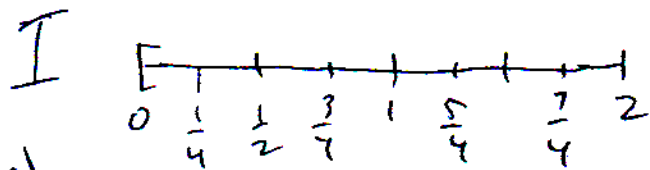
$$\frac{10}{2e} - \frac{10}{e}$$

2-1



85.05 are 62

$$\begin{array}{r} 3.679 \\ 1.837 \\ \hline (1.842) 2 \\ 3.684 \end{array}$$



a)

1) x_i is the sample point of the section.

$$x_i \in \left\{ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4} \right\}$$

2) $f(x_i)$ is the length of the cross sections at each sample point.

$$f(x_i) \in \left\{ \frac{3}{8}, \frac{27}{8}, \frac{75}{8}, \frac{147}{8} \right\}$$

3) $f(x_i)(x_i - x_{i-1})$ is the area of approx rectangle. $\frac{1}{2}$ each time

$$\in \left\{ \frac{3}{16}, \frac{27}{16}, \frac{75}{16}, \frac{147}{16} \right\}$$

4) $(x_i - x_{i-1})$ is the mesh (if it is the width of the sections, in this problem all sections are $\frac{1}{2}$ wide).

1)

$$\text{II } f'(x) = g \quad f'' = -f \quad h = f^2 + g^2$$

$$h = f^2 + (f')^2$$

$$\begin{aligned} h'(x) &= 2f f'(x) + 2f' f'' \\ &= 2fg + 2g(-f) \\ &= 2fg - 2fg \\ &= 0 \end{aligned}$$

so $h'(x) = 0$ ie h is a constant fcn.

Since $h(0) = 5$ then $h(10) = 5$ also

$$\begin{aligned} 6 \cdot \frac{1}{16} &= \frac{6}{16} \\ 3 \cdot \frac{9}{8} &= \frac{27}{8} \\ 3 \cdot 25 &= 75 \\ 49 &= 49 \end{aligned}$$

$$B) \frac{3}{16} + \frac{27}{16} + \frac{75}{16} + \frac{147}{16}$$

$$\frac{3}{16} [1 + 9 + 25 + 49]$$

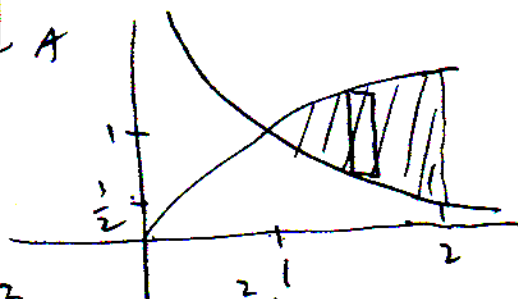
$$C) \frac{3}{16} \sum_{i=1}^4 (2i-1)^2$$

$$D) \int_0^2 6t^2 dt =$$

$$\frac{6(2)^3}{3} - \frac{6(0)^3}{3}$$

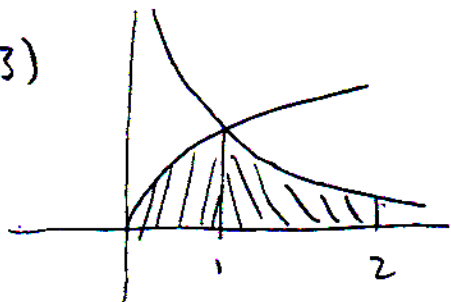
2.8 - 0
16
The object travels 16 feet.

III A



$$\begin{aligned} \int_1^2 \sqrt{x} \cdot \frac{1}{x} dx &= \int_1^2 \frac{1}{\sqrt{x}} dx \\ \frac{2}{3} x^{3/2} \Big|_1^2 &= \ln x \Big|_1^2 \\ \frac{2\sqrt{8}}{3} - \frac{2}{3} &= \ln 2 + \ln 1 \\ \frac{2\sqrt{8}}{3} - \frac{2}{3} &= 4\sqrt{2} - 2 - \ln 2 \end{aligned}$$

III B)

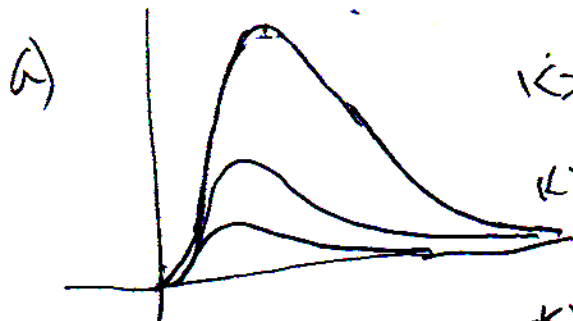


$$\int_0^1 x^{\frac{1}{2}} dx + \int_1^2 \frac{1}{x} dx$$

$$\frac{2}{3} x^{\frac{3}{2}} \Big|_0^1 + \ln x \Big|_1^2 = \frac{2}{3} - 0 + \ln 2 - 0 = \frac{2 + 3 \ln 2}{3}$$

IV $f(x) = 10 x e^{-kx}$

$k=1 (1, 3.679) \quad \left(\frac{1}{k}, \frac{10}{k} e^{-1}\right)$



$k=2 (.5, 1.839)$

$k=3 (.333, 1.228)$

c) as k gets bigger the max pt gets closer to the origin.

d) the max value lies on the y where $\frac{10}{e} x$

b) $f'(x) = e^{-kx} + K x e^{-kx}$
 $e^{-kx} [1 - kx] = 0$ set
 $x = \frac{1}{k}$

$f\left(\frac{1}{k}\right) = 10 \cdot \frac{1}{k} e^{-k\left(\frac{1}{k}\right)}$
 $10 \frac{1}{k} e^{-1}$

k	max	$\frac{10}{e}$
1	$\frac{10}{e}$	Slope is $\frac{10}{e}$
2	$\frac{10}{2e}$	$\lim_{n \rightarrow \infty} \frac{10}{ne} \rightarrow 0$
3	$\frac{10}{3e}$	So y intercept is 0
$\vdots$	$\vdots$	$y = \frac{10}{e} x$
N	$\frac{10}{Ne}$	

EXT 19

$f(x) = \int_0^x \sin^{-1} t dt = \int_0^x \sin^{-1} t dt$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$f'(x) = \sin^{-1} u \cdot du$

$\sin^{-1} \sin x \cos x$

$\sin^{-1}(\sin(\pi-x)) \cos x$

$f'(x) = (\pi-x) \cos x$

But $\frac{\pi}{2} < x < \pi$
 since we are in 2nd quadrant,
 $\sin^{-1}(\sin x) = \sin^{-1}(\sin(\pi-x))$
 so

II

- b) $R(t)$ is continuous and differentiable for $[0, 24]$
 $R(0) = R(24) = 9.6$ $\therefore$ by Rolle's Thm there exists
 ξ such that $R(\xi) = 0$, so yes that point exists.
 Explained but no mention of
 Rolle's Thm = -1
 Example = -5

c) $Q(t) = \frac{1}{79} (768 + 23t - t^2)$

$$\frac{1}{24} \int_0^{24} \frac{1}{79} (768 + 23t - t^2) dt = 10.785$$

water flowed at 10.785 gal per hour.

- III a) ξ_i is sample point in the section $[x_{i-1}, x_i]$. ξ_i is usually
 picked the Left, Right or mid pt. at sections
 b) $x_i - x_{i-1}$ is width of i th section. The largest of these is called
 the mesh. as the mesh $\rightarrow 0$ the error $\rightarrow 0$
 c) $f(\xi_i)$ is the height of the i th sample rectangle. when
 multiplied by the width of the section we get the area
 of the rectangle.
 d) $f(\xi_i)(x_i - x_{i-1})$ area of i th rectangle. These are summed to
 approximate the area under the curve
 e) mesh is the largest of the sections. see details under "b".

Extra.

Let N = number of people in audience

- $.20N$ = heard 60 minutes
 $.10N$ = heard 20 minutes
 $.135N$ = heard 20 min
 $.135N$ = heard 40 min

total N people heard
 $60(.12N) + 0(.10N) + 20(.35N)$
 $+ 40(.35N)$

$$12N + 0 + 7N + 14N$$

33N minutes of
 the speech

so the average minutes heard
 was $\frac{33N}{N} = 33$ minutes.

Copy original problem.

Convince me that you understand the concept!

Chapters 7 & 8 Exam

I

Briefly explain the meaning and use of each of the following terms:

(30 pts total)

- a) X_i b) $(x_i - x_{i-1})$ c) $F(X_i)$ d) telescoping sum

II

Let $F(x) = \frac{1}{x+1}$, $x_i = \frac{i}{n}$, $X_i = \frac{i-1}{n}$ and $i = 1, 2, \dots, n$.The points form a partition of the closed interval $[0, 1]$.

Use summation notation to write the corresponding approximating sum.

While keeping the sigma notation, simplify to remove the letters F , X , and x .

(15 pts)

III

Given the three equations: $x^2 + y^2 = 4$, $y = x + 2$, and $y = x$.

(15 pts total)

- A) Sketch the three equations on the same axis. Shade the region above the x -axis bounded by these curves.
- B) Set up, but Do Not Evaluate the definite integral(s) which describe the shaded region in part A.

IV

Let $F(t)$, $G(t)$, and $H(t)$ be functions such that: $G(t) = F'(t)$ and $H(t) = G'(t)$ on $[0, 1]$. (15 pts total)Suppose $F(0) = 0$, $F(1) = 1$, $G(0) = 0$, $G(1) = 0$, $H(0) = 2$, and $H(1) = 1$

Find and briefly explain your reasoning:

a) $\int_0^1 G(t) dt$ b) $\int_0^1 H(t) dt$

V

Consider the area under the curve $f(x) = x^2$ over the closed interval $[0, 2]$ (25 pts tot)

- a) Draw a clear, labeled, easy to read graph of the situation.
- b) Partition $[0, 2]$ into 4 sections of equal length. List the partition points.
- c) What is the mesh of the partition?
- d) Choose sampling numbers to be the midpoints of each section. List the sampling numbers.
- e) Write an approximating sum using these partition points and sampling numbers. Do not evaluate.
- f) Rewrite part e above using sigma notation.

Extra Credit 5 pts

Consider the following definite integral and its solution: $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{3} \Big|_{-1}^2 = -\frac{3}{8}$

"Since the function $f(x) = x^{-4}$ is positive, the area under it and above the x -axis is positive. The integral should also be positive."

Agree or disagree with the above quote and defend your position.

85 NS 5000

5000

100

97-2

95

74

12-2

89

84-3

80

75

65

57

$$\frac{1201}{15} = 80.06$$

Good exam, most did
a good job. I'm
satisfied.

- I
- ξ_i sample point in section $[x_i - x_{i-1}]$ depending on the function, this changes approx from over to under or vice versa
 - $(x_i - x_{i-1})$ section, The largest section is called the mesh, we take the limit as the mesh $\rightarrow 0$ to get exact area
 - $f(\xi_i)$ height of current rectangle, rectangle area is found by multiplying this height by the width of the sections.
 - telescoping sum, when the summation is expanded the section values add and subtract $x_1 - x_0 + x_2 - x_1 + \dots + x_n - x_{n-1}$ which adds to $x_n - x_0$

II

$$f(x) = \frac{1}{x+1} \quad x_i = \frac{i}{N} \quad \xi_i = \frac{i-1}{N} \quad i = 1, 2, \dots, N$$

$$\sum_{i=1}^N f(\xi_i) (x_i - x_{i-1})$$

$$\sum_{i=1}^N \frac{1}{1 - 1 + N} \cdot \frac{1}{N}$$

$$\sum_{i=1}^N \frac{1}{i-1+N}$$

$$f(\xi_i) = \frac{1}{\frac{i-1}{N} + 1}$$

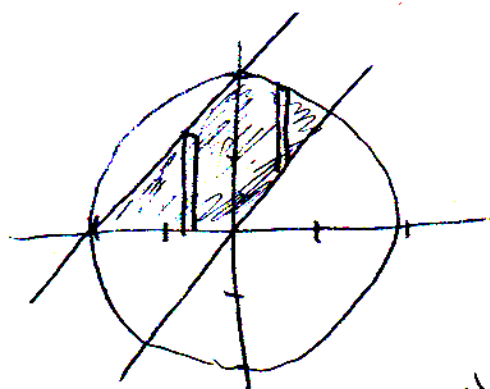
$$= \frac{1}{\frac{i-1+N}{N}}$$

$$= \frac{N}{i-1+N}$$

$$x_{i-1} = \frac{i-1}{N}$$

$$x_i - x_{i-1} = \frac{i}{N} - \frac{i-1}{N} = \frac{1}{N}$$

III



$$\int_{-2}^0 x+2 \, dx + \int_0^{\sqrt{2}} \sqrt{4-x^2} - x \, dx$$

$$\begin{aligned} x^2 + y^2 &= 4 \\ x^2 + x^2 &= 4 \\ 2x^2 &= 4 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

IV

$$\begin{aligned} G(x) &= f'(x) \\ H(x) &= G'(x) \end{aligned}$$

$$f(0) = 0$$

$$f(1) = 1$$

$$G(0) = 0$$

$$G(1) = 0$$

$$H(0) = 2$$

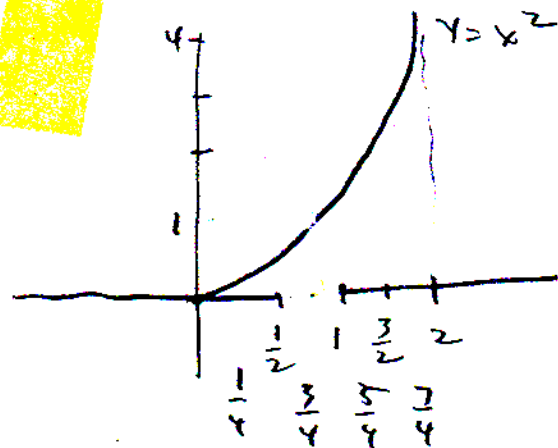
$$H(1) = 1$$

$$a) \int_0^1 G(x) \, dx = F(1) - f(0) = 1 - 0 = 1$$

"the den of what is G"

$$b) \int_0^1 H(x) \, dx = G(1) - G(0) = 0 - 0 = 0$$

V



partitions at $0, \frac{1}{2}, 1, \frac{3}{2}, 2$

mesh = $\frac{1}{2}$

sample pts $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$

$$\left(\frac{1}{4}\right)^2 \cdot \frac{1}{2} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{2} + \left(\frac{5}{4}\right)^2 \cdot \frac{1}{2} + \left(\frac{7}{4}\right)^2 \cdot \frac{1}{2}$$

$$\frac{1}{2} \sum_{i=1}^4 \left(\frac{2i-1}{4}\right)^2 = \frac{1}{32} \sum_{i=1}^4 (2i-1)^2$$

Extra $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{3} \Big|_{-1}^2 = -\frac{3}{8}$

Since $f(x) = \frac{1}{x^4}$ is not continuous in the interval $[-1, 2]$ the $-\frac{3}{8}$ has no meaning. Certainly the area is positive - it is infinite!

Chapters 7 & 8 Exam

I

Use the ~~definition of definite integral~~ to show that $\int_2^6 dx = 12$.

(hint: The definition has nothing to do with a graph.)

II

Differentiate: $g(x) = \int_{x^2}^2 \sqrt{\cos t} dt + \int_2^3 \sqrt{\cos t} dt$

$$g(x) = \int_{x^2}^2 \sqrt{\cos t} dt + \int_2^3 \sqrt{\cos t} dt$$

III

Given the three equations: $x^2 + y^2 = 4$, $y = x + 2$, and $y = x$.

(20 pts total)

A) Sketch the three equations on the same axis. Shade the region above the x-axis bounded by these curves.

B) Set up, but Do Not Evaluate the definite integral(s) which describe the shaded region in part A. Give a very explicit explanation of your methods.

IV

The value $\frac{1}{b-a} \int_a^b f(x) dx$ is called the mean (or average) value of f on the interval $[a, b]$ and is usually denoted f_{av} . Let $f(x) = \sin x^2$ for $0 \leq x \leq \sqrt{\pi}$

(20 pts total)

- Describe the window setting necessary to yield a clear view of the function in the given interval on your calculator.
- Sketch your graph.
- Predict the mean value, f_{av} , of f on $[0, \sqrt{\pi}]$. Call your prediction A.
- Use the program RiemannC with $n = 100$ to find the approximation of the integral. Compute f_{av} using this value and call this value B.
- Use fnInt ($Y_1, X, 0, \sqrt{\pi}$) / $\sqrt{\pi}$ (found in the MATH menu). Call this value C.
- Compare the values A, B, and C. Briefly explain.

V

The velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown in a table of values for $v(t)$, at 5 second intervals of time.

Approximate the definite integral, $\int_0^{50} v(t) dt$ with a riemann sum, using the mid-

points of five subintervals of equal length. Using correct units, explain the meaning of this integral. (20 pts)

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

VI

Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$. Draw the graph showing the area computed and compute the exact area described. (tot 20)

a) $\int_1^2 f(x) dx - \int_1^2 g(x) dx$

b) $\int_0^1 f(x) dx + \int_1^2 g(x) dx$

Extra Credit 5 pts

Lemma 3, used by our text in the proof of the Fundamental Theorem of Calculus states:

$$f(X)(b-a) = \int_a^b f(x) dx. \text{ Using the function in Section IV. Compute the "cap X" value.}$$

100	0
90	4
80	3
70	3
60	2
50	1

97

54

$$\begin{array}{r} 1030 \\ 13 \end{array}$$

79.2

Question

I had an error on the

$$\cos 2 < 0$$

$$\cos 3 < 0$$

It should have been

something like

$$\int_{-0.73}^{0.73} \sqrt{\cos t} dt + \int_{0.73}^{1.4} \sqrt{\cos t} dt$$

$$I \int_2^4 6 dx = 12$$

$$W 2 = x_0 < x_1 < x_2 \dots < x_n = 4$$

$$\sum_{i=1}^n \bar{x}_i (x_i - x_{i-1})$$

use of
FTC
= -4

$$\sum_{i=1}^n 6 (x_i - x_{i-1})$$

$$6(4-2)$$

$$6 \cdot 2$$

$$12$$

$$(0, 2)$$

$$y = x + 2$$

$$y = x$$

$$(\sqrt{2}, \sqrt{2})$$

$$(-2, 0)$$

III

$$x^2 + x^2 = 4$$

$$2x^2 = 4$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

"this y value minus this x value"

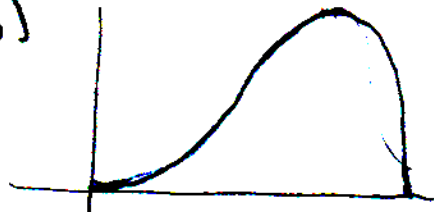
V

$$\int_0^{50} v(t) dt$$

$$IV \frac{1}{b-a} \int_a^b f(x) dx = f_{ave}$$

$$a) x \in [0, \sqrt{\pi}] \quad y \in [0, 1]$$

b)



c) about $\frac{1}{2}$

$$D) \frac{.8947}{\sqrt{\pi}} = .5048$$

$$E) \frac{.894831}{\sqrt{\pi}} = .5049$$

f) The exact value is slightly larger than approx since $N=100$ is approx and $N \rightarrow \infty$ is integral.

$$g(x) = \int_x^2 \sqrt{\cos t} dt + \int_2^3 \sqrt{\cos t} dt$$

$$g(x) = \int_x^2 \sqrt{\cos t} dt = \text{constant}$$

$$g'(x) =$$

$$= -2x \sqrt{\cos x^2}$$

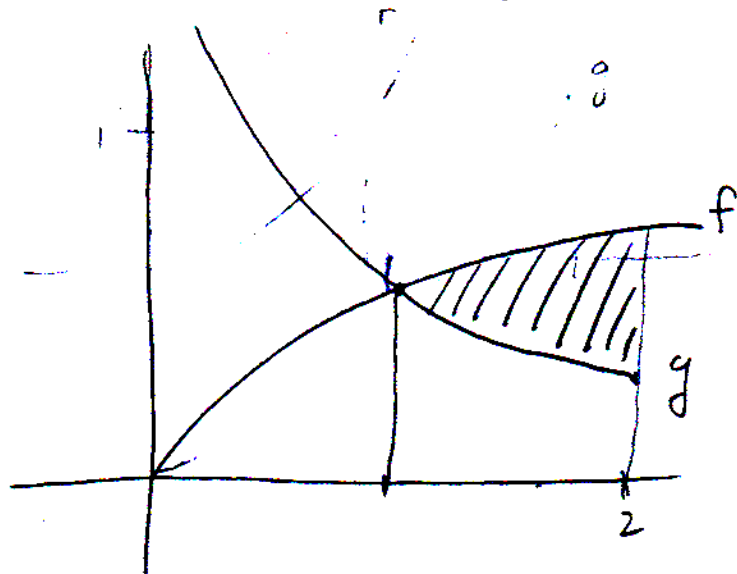
$$\int_{-2}^0 x+2 dx + \int_0^{\sqrt{2}} \sqrt{4-x^2} - y dy$$

$$10(12) + 10(30) + 10(70) + 10(81) + 10(60)$$

$$\begin{array}{r} 120 \\ 300 \\ 700 \\ 810 \\ 600 \\ \hline 2530 \end{array}$$

This integral is the total distance traveled in feet during the time from zero to 50 seconds.

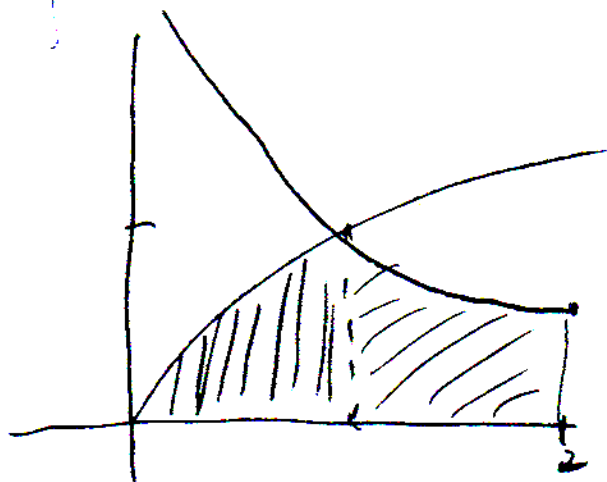
VT $f(x) = \sqrt{x}$ $g(x) = \frac{1}{x}$



$$\int_1^2 \sqrt{x} dx - \int_1^2 \frac{1}{x} dx$$

$$\frac{2}{3} x^{\frac{3}{2}} \Big|_1^2 - (\ln 2 - \ln 1)$$

$$\frac{4\sqrt{2}}{3} - \frac{2}{3} - \ln 2 = .526$$



$$\int_0^2 \sqrt{x} dx + \int_1^2 \frac{1}{x} dx$$

$$\frac{2}{3} x^{\frac{3}{2}} \Big|_0^2 + \ln 2$$

$$\frac{4\sqrt{2}}{3} + \ln 2 = 1.36$$

EX 7.14

$$f(x) (b-a) = \int_a^b f(x) dx$$

$$f(x) (b-a) = \int_a^b \sin^2 x^2 dx$$

$$f(x) = \frac{1}{\sqrt{\pi}} \quad (.89485...)$$

$$\sin^2 x^2 = .50485$$

$$x^2 = .52921$$

$$x = .7275$$

Advanced Placement Calculus

Copy original problem.

Convince *me* that *you* understand the concept!

No Calculators.

Name

Per

Date

Chapters 1 - 8 Exam

I

The definite integral of a function f over an interval $[a, b]$ is denoted by $\int_a^b f(x) dx$ and is defined as follows: $\int_a^b f(x) dx = \lim_{\text{mesh} \rightarrow 0} \sum_{i=1}^n f(X_i)(x_i - x_{i-1})$. ~~Describe each of the following items and describe its purpose in the context of our class.~~ *Keep* (4 pts ea)

~~a) Explain in general terms what each of the following items is and its numeric value(s) as used in this problem.~~

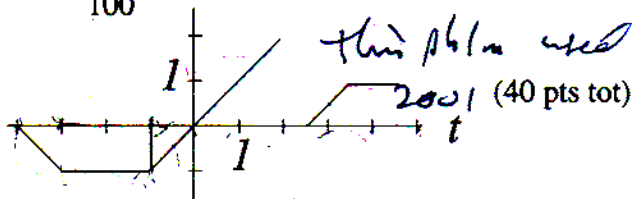
- 1) X_i 2) $(x_i - x_{i-1})$ 3) $f(X_i)$ 4) $f(X_i)(x_i - x_{i-1})$ 5) mesh

II

Write the following in summation notation: $\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots + \frac{1}{100}$ (5 pts)

III

Let $g(x) = \int_{-1}^{2x} f(t) dt$ when given the graph of $y = f(t)$:



- A) Explain clearly why the domain is: $D_g: -2 \leq x \leq 1$. Include reasoning how you know there is not a second interval.
B) For what values of x does $g'(x) = 1$?
C) Determine the coordinates of all extrema and all axis intercepts.
D) Draw $g(x)$.

IV

Find the second derivative with respect to x evaluated at $x = 3$ for each of the following: (25 pts tot)

a) $\int_0^x \sin t^2 dt$

b) $\int_0^{\sin x} \sin^{-1} t dt$

c) $\int_1^2 \ln(1+t^2) dt$

d) $\int_t^x \frac{1}{t} dt$

V

Consider the following definite integral and its solution: $\int_{-1}^2 \frac{1}{x^4} dx = -\frac{x^{-3}}{3} \Big|_{-1}^2 = -\frac{3}{8}$ (10 pts)

"there is such a thing as negative area and this is an example of it."
~~"Since the function $f(x) = \frac{1}{x^4}$ is positive for all x , the area under it and above the x -axis should also be positive."~~

Agree or disagree with the above quote and defend your position.

Extra Credit ----- 5 pts -----

Find the coordinates of the focus of the parabola which has vertex at $(1, -2)$ and passes through the point $(3, 6)$.

2/19/98

100	0	93
90	2	
80	4	
70	3	
60	5	
59	2	56

$$\frac{1195}{16} = 74.7$$

a few cosmetic
errors as marked.
change the quote
as indicated.

- I
- x_i The sample point which in the section $x_i - x_{i-1}$. This is the x value the function will act upon.
 - $x_i - x_{i-1}$ width of the i^{th} section. This is the width of the approximating rectangle. It is the operational part of telescoping sums.
 - $f(x_i)$ height of the i^{th} approximating rectangle. Used to find the area of the approximating rectangle.
 - $f(x_i)(x_i - x_{i-1})$ The area of the i^{th} approx rectangle. These are summed to approximate the area under the curve. When the limit is applied to this sum, the area becomes exact.
 - nish is the width of the widest section. As the mesh goes to zero the section $x_i - x_{i-1}$ also shrinks toward zero. The final result is the exact area under the curve.

II

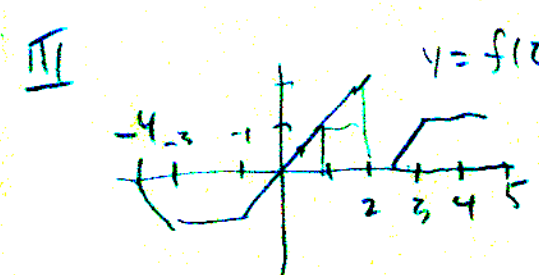
$$\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots + \frac{1}{100}$$

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{50} \right)$$

$$\frac{1}{2} \sum_{i=2}^{50} \frac{1}{i}$$

a) $-4 \leq t \leq 2$
 $-4 \leq 2x \leq 2$
 $-2 \leq x \leq 1$

$\frac{5}{2} \leq t \leq \frac{9}{2}$
 $\frac{5}{2} \leq 2x \leq \frac{9}{2}$
 $\frac{5}{4} \leq x \leq \frac{9}{4}$

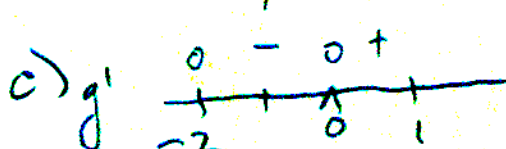


$$g(x) = \int_1^{2x} f(t) dt$$

(B) $g'(x) = f(2x) \cdot 2 \stackrel{\text{Set}}{=} 1$
 $f(2x) = \frac{1}{2}$

$2x \in \frac{1}{2}$ (from graph)
 $x = \frac{1}{4}$

Does not include $t = -1$
 $\therefore$ not part of Domain
 by FTC, f is not continuous



max $(-2, \frac{1}{2})$ End at dec limit.
 min $(0, \frac{1}{2})$ f' test. $\therefore$ interval
 max $(1, \frac{3}{2})$ $\leftarrow$ would inc to it.
 $(\frac{1}{2}, 0)$ $\leftarrow$ interval $(\frac{1}{2}, 0)$

$g'(x) = 2f(2x) \stackrel{\text{Set}}{=} 0$
 $f(2x) = 0$
 $2x = 0 \quad 2x = -4$
 $x = 0 \quad x = -2$

$g(-2) = \int_{-1}^{-4} f(t) dt = - \int_{-4}^{-1} f(t) dt = -(-2.5) = 2.5$

$g(0) = \int_{-1}^0 f(t) dt = -\frac{1}{2}$

$g(-\frac{1}{2}) = \int_{-1}^{-1} f(t) dt = 0$

$g(\frac{1}{2}) = \int_{-1}^1 f(t) dt = 0$

$g(1) = \int_{-1}^2 f(t) dt = -\frac{1}{2} + 2 = \frac{3}{2}$

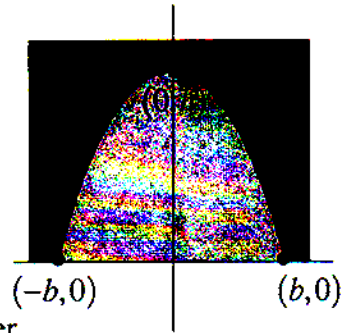


This “honor quiz” is to be done by you ***totally on your own***.

You are not allowed to discuss your solution with anyone until the quiz has been turned in Tuesday morning, February 11, not later than 7:55 in the morning. The value of this quiz is 3 homework assignments. Answer the questions on this paper. You may use the back if necessary. No late submissions!

An arched window with base width $2b$ and height h is to be set into a wall. The arch is to be either an arc of a parabola or a half-cycle of a cosine curve.

- a) If the arch is an arc of a parabola, write an equation for the parabola relative to the coordinate system shown in the figure. Explain.
- b) If the arch is a half-cycle of a cosine curve, write an equation for the cosine curve relative to the coordinate system shown in the figure. Explain.
- c) Of these two window designs, which has the greater area? Justify your answer.



RiemannC

Program for TI 82/83

```

1 → L
2 → R
FnOff
Prompt A
Prompt B
Lbl 1
Prompt N
(B-A)/N → H
Disp "L(1) OR R(2)"
Input W
0 → S
ClrDraw
DrawF Y1
If W ≠ L
Goto 2
N-1 → T
0 → V
H → Z
Lbl 3
A+VH → X
Y1 → Q
S+QH → S
X → P
Line(P,0,P,Q)
Line(P,Q,P+Z,Q)
Line(P+Z,Q,P+Z,0)
IS>(V,T)
Goto 3
Pause
Disp "SUM"
Disp S
Pause
Goto 1
Lbl 2
If W ≠ R
Goto 1
N → T
1 → V
-H → Z
Goto 3

```

1. Key in the program for your machine.

This is zero → 0

IS>(is a program control word found on the 82 under "Program CTL"

The 82 only allows single letter variable names so VH means V times H.

On the 85 you must explicitly state the multiply.

2. Set $Y_1 = x^2$ (on the 85 use the x-var key to make the x for this definition.)

A = 0

B = 3

3. Set the window: 0,3,1,0,9,3

4. Copy the following table to your paper:

Try #	N	L(1)R(2)	Result
1	6	1	6.875
2	6	2	
3	12	1	
4	12	2	
5	18	1	
6	18	2	
7	100	1	
8	100	2	
9	1000	1	
10	1000	2	9.0135

5. Run the program RiemannC and record your results in your table.

TI 85/86/89 Program

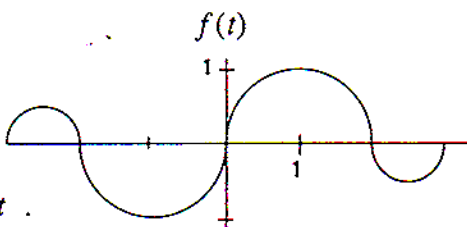
```

1 → I
2 → R
FnOff
Prompt a
Prompt b
Lbl o
Prompt n
(b-a)/n → h
Disp "l(1) OR r(2)"
Input w
0 → s
ClrDraw
DrawFunc y1(x)
If w ≠ 1
Goto d
n-1 → t
0 → v
h → z
Lbl k
a+v*h → x-var key
y1(x) → q
s+q*h → s
x-var key → p
Line p,0,p,q
Line p,q,p+z,q
Line p+z,q,p+z,0
If v < t Then
v+1 → v
Goto k
Else
Pause
Disp "sum"
Disp s
EndIf
Pause
Goto o
Lbl d
If w ≠ r
Goto p
n → t
1 → v
-h → z
Goto k
EndPrgm

```

The graph of the function f on the right consists of four semicircles.

Given: $g(x) = \int_0^x f(t) dt$, $h(x) = \int_1^x f(t) dt$ and $j(x) = \int_0^{2x} f(t) dt$.



- I
 - a) Consider a circle of radius one-half. What is the area of its semicircle?
 - b) Consider a circle of radius one. What is the area of its semicircle?
 - c) Is the function $f(t)$ odd, even, or neither?

- II
 - a) How does the answer to part (Ic) effect $g(x)$?
 - b) Find: $g(0)$, $g(2)$, $g(3)$, $g(-2)$, $g(-3)$
 - c) Make a number line for $g'(x)$. Mark zeros as well as indicate positive and negative intervals.
 - d) Specifically state the intervals when $g(x)$ is increasing; decreasing.
 - e) Determine the *coordinates* of all extrema of $g(x)$. Justify, of course.
 - f) Determine the *coordinates* of all Points of Inflection of $g(x)$.
 - g) Sketch $g(x)$.
 - h) For what values of x is $g(x)$ non-negative.

- III
 - a) What is the domain of $h(x)$?
 - b) Compute: $h(1), h(0), h(2), h(3), h(-2), h(-3), h(-1)$
 - c) Draw a $h'(x)$ number line. Indicate $+$, $-$ as appropriate.
 - d) Compute *coordinates* of extrema. Justify, of course.
 - e) Draw a $h''(x)$ number line. Indicate $+$, $-$ as appropriate.
 - f) Compute *coordinates* of PI.
 - g) Sketch $h(x)$.

- IV
 - a) What is the domain of $j(x)$?
 - b) As in section III, compute the significant values of $j(x)$.
 - c) Draw $j'(x)$ number line, Indicate $+$, $-$ as appropriate.
 - d) Compute *coordinates* of extrema. Justify, of course.
 - e) Draw $j''(x)$ number line, Indicate $+$, $-$ as appropriate.
 - f) Compute *coordinates* of PI.
 - g) Sketch $j(x)$.

* Ask your parents about Ward Bond's famous words.

I a) $\frac{\pi}{8}$

I b) $\frac{\pi}{2}$

I c) f is an odd function because the function reflects through the origin.
For example, $(1,1)$ reflects to $(-1,-1)$.

II a) g will be an even function so $g(a) = g(-a)$.

II b) $g(0) = 0$ because $g(0) = \int_0^0 f(t) dt$.

$$g(2) = \int_0^2 f(t) dt \text{ which is } \frac{\pi}{2} \text{ (from part b) so } g(-2) = g(2) = \int_0^2 f(t) dt = \frac{\pi}{2}$$

$$g(3) = \int_0^3 f(t) dt = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8} \text{ subtracting the smaller semicircle from the larger so}$$

$$g(-3) = g(3) = \int_0^3 f(t) dt = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8}.$$

II c) $\frac{1+0-0+0-1}{-3-2 \quad 0 \quad 2 \quad 3}$

II d) $g'(x) > 0$ for $-3 < x < -2$ and for $0 < x < 2$ thus g is increasing in these intervals.
 $g'(x) < 0$ for $-2 < x < 0$ and for $2 < x < 3$ thus g is decreasing in these intervals.

II e) $g(-3) = g(3) = \frac{3\pi}{8}$. The left end point is a minimum because g increases from it and the right end point

is a minimum because g decreases to it. coordinates: $\left(3, \frac{3\pi}{8}\right)$ and $\left(-3, \frac{3\pi}{8}\right)$

$g(-2) = g(2) = \frac{\pi}{2}$ are maximums by $f'(x)$ test. coordinates: $\left(2, \frac{\pi}{2}\right)$ and $\left(-2, \frac{\pi}{2}\right)$

$g(0) = 0$ by $f'(x)$. coordinates $(0, 0)$

II f) PI occur in g when $g''(x)$ changes sign. This is when $f'(x)$ changes sign. From the our graph, this is when f changes from increasing to decreasing (or vice versa).

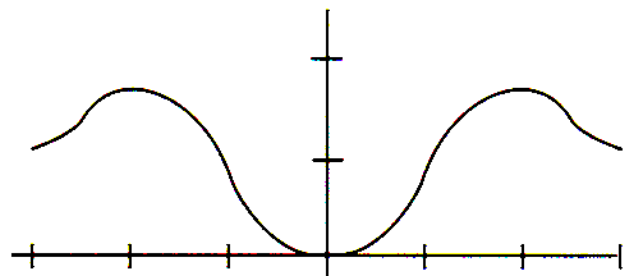
PI occur when x is an element of the set: $\{-2.5, -1, 1, 2.5\}$. Note: $\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$ and $\frac{7\pi}{16} = \frac{\pi}{2} - \frac{\pi}{16}$

Coordinates: $\left(-2.5, \frac{7\pi}{16}\right), \left(-1, \frac{\pi}{4}\right), \left(1, \frac{\pi}{4}\right), \left(2.5, \frac{7\pi}{16}\right)$

$$\begin{array}{cccccccc} \text{up} & \text{dn} & & \text{up} & & \text{dn} & \text{up} & \\ 1 & + & 0 & - & 0 & + & 0 & - & 0 & + & 1 \\ -3 & -2.5 & -2 & -1 & 0 & 1 & 2 & 2.5 & 3 \end{array}$$

Note: $g''(x)$ DNE for $x \in \{-2, 0, 2\}$ because the tangent line to the graph of f becomes vertical.

II g)



II h) g is non-negative for all $x \in [-3, 3]$.

III a) $D_h x \in [-3, 3]$ There is no change because upper bound is x (rather than $x+1$, for example)

III b) $h(1) = \int_1^1 f(t) dt = 0 \quad (1, 0) \quad h(0) = \int_1^0 f(t) dt = -\int_0^1 f(t) dt = -\frac{\pi}{4} \quad \left(0, -\frac{\pi}{4}\right)$

$h(2) = \int_1^2 f(t) dt = \frac{\pi}{4} \quad \left(2, \frac{\pi}{4}\right) \quad h(3) = \int_1^3 f(t) dt = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8} \quad \left(3, \frac{\pi}{8}\right)$

$h(-1) = \frac{\pi}{4} - \frac{\pi}{4} = 0 \quad (-1, 0) \quad h(-2) = -\left(-\frac{\pi}{4}\right) = \frac{\pi}{4} \quad \left(-2, \frac{\pi}{4}\right)$

$h(-3) = \text{this is } h(-2) - \frac{\pi}{4} = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8} \quad \left(-3, \frac{\pi}{8}\right)$

III c) This is the same as it was for II c because $h'(x) = f(x)$ just as $g'(x) = f(x)$.

$$\begin{array}{ccccccc} 0 & + & 0 & - & 0 & + & 0 & - & 0 \\ -3 & -2 & 0 & 2 & 3 \end{array}$$

III d) $\left(-3, \frac{\pi}{8}\right)$ min. end pt increasing from it. $\left(-2, \frac{\pi}{4}\right)$ max $f'(x)$ test

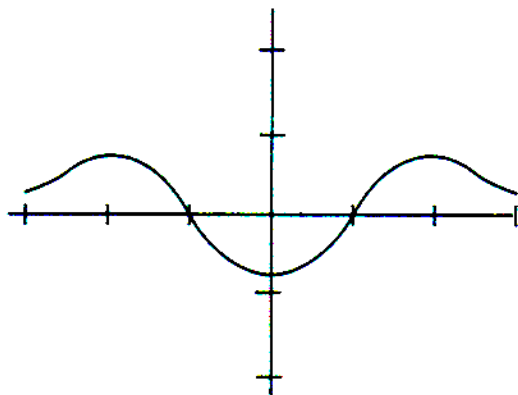
$\left(0, -\frac{\pi}{4}\right)$ min $f'(x)$ test $\left(2, \frac{\pi}{8}\right)$ max $f'(x)$ test $\left(3, \frac{\pi}{8}\right)$ min end pt decrease to it.

III e) $h''(x)$ $\begin{array}{ccccccc} \text{up} & \text{dn} & & \text{up} & \text{dn} & \text{up} \\ [+ & 0 & - & 0 & + & 0 & - & 0 & +] \\ -3 & -2.5 & -2 & -1 & 0 & 1 & 2 & 2.5 & 3 \end{array}$ Note: This is exactly the same as $g''(x)$ (II f)

III f) As in part II f, the x values of the PI are the same.

$-\frac{\pi}{4} + \frac{\pi}{2} - \frac{\pi}{16} = \frac{3\pi}{16} \quad \left(-\frac{5}{2}, \frac{3\pi}{16}\right) \quad \frac{\pi}{4} - \frac{\pi}{16} = \frac{3\pi}{16} \quad \left(\frac{5}{2}, \frac{3\pi}{16}\right) \quad (-1, 0), (1, 0)$

III g) $h(x)$:



IV a) $-3 \leq t \leq 3$ so
 $-3 \leq 2x \leq 3$ $D_f x \in \left[-\frac{3}{2}, \frac{3}{2}\right]$
 $-\frac{3}{2} \leq x \leq \frac{3}{2}$

IV b) The $2x$ just changes the period of the function much in the same manner as $\sin x$ compared to $\sin 2x$.

$$j(0) = 0, j(-1) = j(1) = \int_0^{2 \cdot 1 = 2} f(t) dt = \frac{\pi}{2}, j\left(-\frac{3}{2}\right) = j\left(\frac{3}{2}\right) = \int_0^{2 \cdot \frac{3}{2} = 3} f(t) dt = \frac{3\pi}{8}$$

$$j\left(-\frac{1}{2}\right) = j\left(\frac{1}{2}\right) = \int_0^{2 \cdot \frac{1}{2} = 1} f(t) dt = \frac{\pi}{4} \quad j\left(-\frac{5}{4}\right) = j\left(\frac{5}{4}\right) = \int_0^{2 \cdot \frac{5}{4} = \frac{5}{2}} f(t) dt = \frac{\pi}{2} - \frac{\pi}{16} = \frac{7\pi}{16}$$

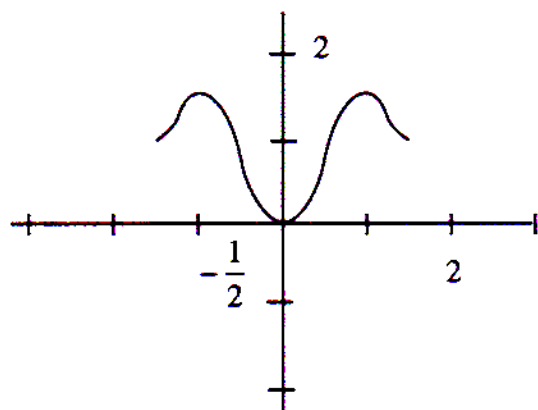
IV c) $j'(x)$ $\frac{[+0-0+\quad 0-]}{-\frac{3}{2} \quad -1 \quad \underbrace{0} \quad \frac{1}{4} \quad 1 \quad \frac{3}{2}}$ $j'\left(\frac{1}{4}\right)$ would be the slope of $f(t)$ at $x = \frac{1}{2}$.
 From the original picture of f , f is clearly increasing. The $+$, $-$ signs alternate from there.

IV d) $\left(-\frac{3}{2}, \frac{3\pi}{8}\right)$ is min. end pt, increasing from it $\left(-1, \frac{\pi}{2}\right)$ is max. f' test. $(0, 0)$ min f' test.
 $\left(1, \frac{\pi}{2}\right)$ max, f' test. $\left(\frac{3}{2}, \frac{3\pi}{8}\right)$ is min. end pt. decreasing to it.

IV e) $\frac{\text{up} \quad \text{dn} \quad \text{up} \quad \text{dn} \quad \text{up}}{[+0-0+0-0+]} \quad \frac{\text{up} \quad \text{dn} \quad \text{up}}{-\frac{3}{2} \quad -\frac{5}{4} \quad -1 \quad -\frac{1}{2} \quad \underbrace{0} \quad \frac{1}{2} \quad 1 \quad \frac{5}{4} \quad \frac{3}{2}}$

IV f) The coordinates are listed in IV b.

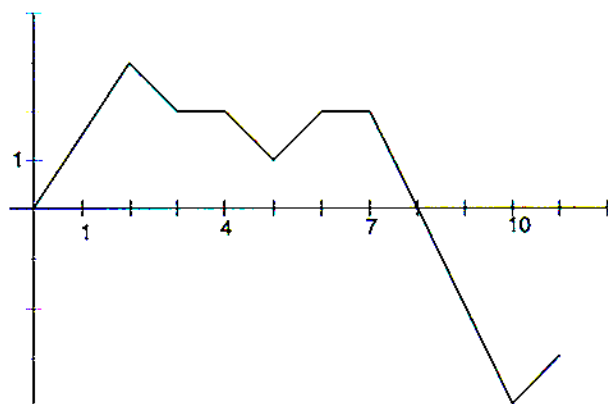
IV g)



Translating Graphenese to English

I

A bug is crawling along a straight wire. The velocity, $v(t)$, of the bug at time t , $0 \leq t \leq 11$, is given in the graph on the right.

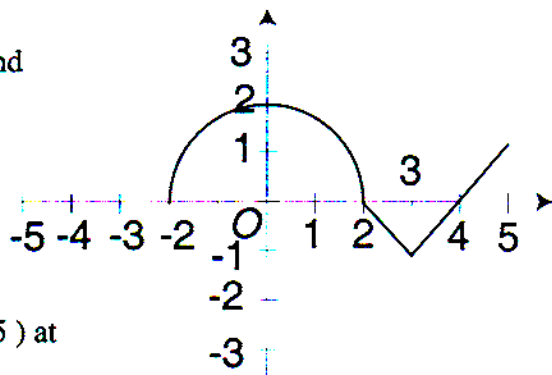


- According to the graph, at what time t does the bug change direction?
- According to the graph, at what time t is the speed of the bug greatest?
- How far did the bug travel?
- How far is the bug from its starting point?
- Why are the answers to parts "c" and "d" different?
- Compute: $\int_0^8 v(t) dt$, $\int_8^{11} v(t) dt$, and $\int_0^{11} v(t) dt$

II

The graph of a function f consists of a semicircle and two line segments as shown. Let g be the function

given by $g(x) = \int_0^x f(t) dt$.

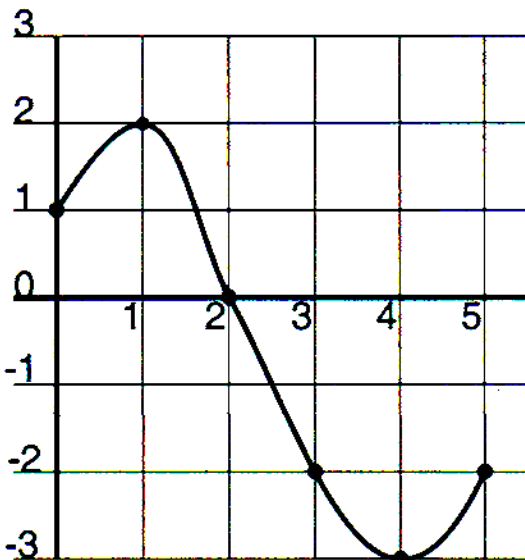


- Find $g(3)$.
- Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.
- Write an equation for the line tangent to the graph of g at $x = 3$.
- Find the x -coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

III

Let f be a function whose domain is the closed interval $[0, 5]$. The graph of f is shown on the right.

Let $h(x) = \int_0^{\frac{x}{2}+3} f(t) dt$



- Find the domain of h .
- Find $h'(2)$.
- At what x is $h(x)$ a minimum? Show the analysis that leads to your conclusion.

I a) The bug changes direction at $t=8$, that is when velocity changes from positive to negative.

b) The speed is the greatest when $t=10$. The speed is -4 . The next fastest was at $t=2$ when the speed was 3 .

c) distance is the integral of velocity so compute the areas between each curve and the x -axis.

above $\frac{1}{2} \cdot 2 \cdot 3 + \frac{(\frac{2+3}{2}) \cdot 1 + 2 \cdot (\frac{1+2}{2}) \cdot 1 + \frac{1}{2} \cdot 2 \cdot 1}{3 + \frac{5}{2} + 4 + 3 + 1} = 13\frac{1}{2}$

below $\frac{1}{2} \cdot 2 \cdot 4 + \frac{1}{2} \cdot (4+3) \cdot 1$
 $4 + \frac{7}{2} = 7\frac{1}{2}$

d) Total distance traveled was $13\frac{1}{2} + 7\frac{1}{2} = 21$

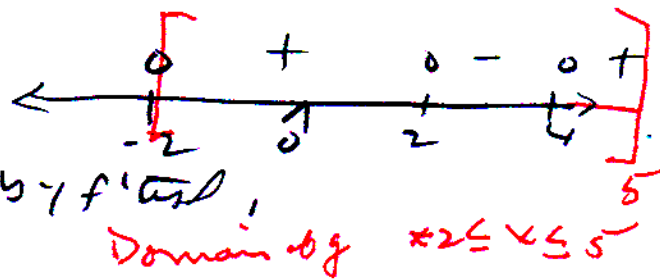
e) $13\frac{1}{2} - 7\frac{1}{2} = 6$ The Bug is 6 from the starting point.

f) we subtract in part e because the bug was returning.
Thus $\int_0^{10} v(t) dt = 6$ net accumulation.

II

a) $g(3) = \int_0^3 f(x) dx = \text{area under } \frac{1}{4} \text{ circle radius } 2 = \pi$
minus area of isosceles $\Delta = \frac{1}{2}$
Since it is below x-axis

b) g will have a max, min or PI at critical values of x .
This is a zero of $g'(x)$ that is when $f(x) = 0$
 f is zero at $-2, 2$ and 4



The only max is at $x=2$ by f' test.

c) Tangent line at $x=3$ has point $(3, \pi - \frac{1}{2})$

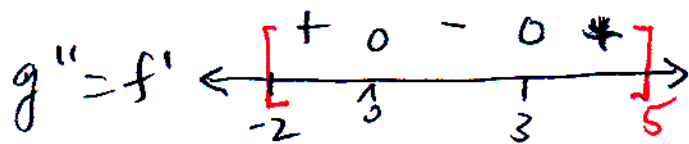
$$m = g'(3) = f(3) = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - (\pi - \frac{1}{2}) = -1(x - 3)$$

$$y = -x + 3 + \pi - \frac{1}{2}$$
$$= -x + \frac{5}{2} + \pi$$

d) a point of inflection occurs when there is a change in concavity. g changes concavity when f' changes sign.



From given graph slope of f is positive $-2 < x < 0$ concave up.

PI at $x=0$ concave up to down
 $x=3$ concave down to up

Note: corners at $x=2$ and $x=3$. f'' exists on continuity not required only change in concavity.
See pg 212

III

a) $0 \leq \frac{x}{2} + 3 \leq 5$

$-3 \leq \frac{x}{2} \leq 2$

$-6 \leq x \leq 4$

$D_h -6 \leq x \leq 4$

because $h'(2)$ from part b was < 0

b) $h'(2) = ?$

$h(x) = \int_0^{\frac{x}{2}+3} f(t) dt \therefore h'(x) = f(\frac{x}{2}+3) \cdot \frac{1}{2}$

$h'(2) = \frac{1}{2} f(\frac{2}{2}+3) = \frac{1}{2} f(4)$

$\frac{1}{2} f(4) \stackrel{\text{from graph}}{=} \frac{1}{2} (-3) = -\frac{3}{2}$

c) $h'(x) \stackrel{\text{set}}{=} 0$

$\frac{1}{2} f(\frac{x}{2}+3) \stackrel{\text{set}}{=} 0$

$f(\frac{x}{2}+3) = 0$

$\frac{x}{2} + 3 \stackrel{\text{set}}{=} 2$

$\frac{x}{2} = -1$

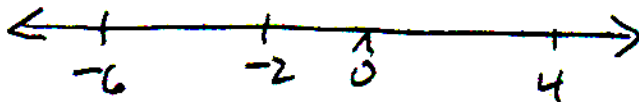
$x = -2$

Reading from graph when $f(x) = 0$

this is similar to

$\sin 2x > \frac{1}{2} \rightarrow 2x \in \{ \dots, \frac{\pi}{6}, \frac{5\pi}{6}, \dots \}$

+ 0 -



Not meaning

$h'(x) = \frac{1}{2} f(\frac{x}{2}+3)$
 $h'(0) = \frac{1}{2} f(3)$
 $= \frac{1}{2} (-2)$
 $= -1 < 0$

$h(x)$ has a minimum at $x = -6$. END point and fctn INCREASES from it,
 ALSO a min at $x = 4$ because fctn DECREASES to it.

Calculus

f, f', f'' (un)ctions

J Mumaugh

Each of the following columns contains four related functions in random order, the graphs of $f(x)$, $f'(x)$, $f''(x)$, and the function $F(x)$ (whose derivative is $f(x)$ ie. $F'(x) = f(x)$). Label each graph with the correct function.

